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A COMPARISON OF PCTR AND CRITICAL EXPERIMENT DETERMINATIONS OF k_∞ FOR TWO WEIGHT PER CENT URANIUM-235 ENRICHED UF_4 IN PARAFFIN

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INTRODUCTION

Nuclear safety specifications for safe handling and storage of fissile materials requires adequate knowledge of the criticality parameters of these materials. One of the most useful of these parameters is the infinite medium thermal neutron multiplication factor commonly referred to as k_∞.

The Physical Constants Test Reactor is a useful tool for determining this parameter; however, the techniques of k_∞ measurements in the PCTR have not always been fully understood. For this reason an experiment was designed to check the value of k_∞ obtained from PCTR measurements. The method was to obtain an independent value of k_∞ from critical experiments on the same material used in the PCTR experiments.

SUMMARY

The value of k_∞ for 2 w/o U^{235} enriched UF_4 paraffin moderated at a H/U^{235} atomic ratio of 195 was determined in the PCTR.

This material was originally obtained from Oak Ridge National Laboratories to which it was returned in order that critical experiments might be conducted on this material to determine k_∞ in this manner. The two values of k_∞ were then compared.

The value of k_∞ as determined in the PCTR was dependent upon the cross-section values which were chosen. In this experiment "effective" cross-section values obtained by averaging the cross section over the Wigner-Wilkens spectrum of a similar type of mixture were used. (9)

The value of k_∞ determined in the PCTR was 1.216 ± 0.013. The value obtained from the work carried out at the Oak Ridge National Laboratory was 1.200 ± 0.011 from a one-group treatment of the critical experiments and 1.202 ± 0.012 from a two-group treatment of the work. (12)

A theoretical calculation of k_∞ is shown in the Appendix and gives a k_∞ value of 1.23. This was in good agreement with the experimental value.

Critical experiments done on this material at Oak Ridge National Laboratory show that the minimum critical mass for a bare "square" cylinder contains approximately 28 kg of U^{235}.[2] A calculated value of the amount of U^{235} in a just critical bare square cylinder was obtained using the experimental k_∞ value. Again there was good agreement with the experimental results. This calculation is shown in the Appendix.

DISCUSSION

The infinite medium thermal neutron multiplication factor, hereafter referred to as k_∞ of a multiplying material may be determined from the quantity of thermal neutron absorber necessary to reduce k_∞ of the multiplying medium to unity.[3, 4]

The quantity of thermal neutron absorber necessary may be determined from the principle that k_∞ of the infinite just critical medium and a vacuum are the same, namely, unity. Then the substitution of a void for a finite region of the material will produce no perturbation on the system.[3] The necessary amount of thermal neutron absorber, hereafter referred to as the poison, to produce this condition may then be determined. The assumption is made that the addition of the poison to the medium changes only the thermal utilization of the medium.[3, 4] The validity of this assumption will be shown in later arguments.

The infinite medium thermal neutron multiplication factor for the unpoisoned system is given by

$$k_\infty = \eta \epsilon p f$$

where these terms have their usual meaning.[1]
Then k_∞ for the poisoned system is given as

$$k'_\infty = \eta \epsilon p f' = 1$$

then defining Δk_∞ as

$$\Delta k_\infty = k_\infty - 1$$

Δk_∞ is then found to be

$$\Delta k_\infty = k_\infty - 1 = \frac{k_\infty - k_\infty'}{k_\infty'} = \frac{f - f'}{f'}$$

The thermal utilization, f, for a homogeneous medium is defined as

$$f = \frac{(\Sigma_a \phi V)_{fuel}}{\sum_i (\Sigma_a \phi V)_i}$$

where "fuel" refers to uranium and "i" refers to all the other materials in the system except the poison

Then

$$\Delta k_\infty = \frac{f - f'}{f'} = \frac{\dfrac{(\Sigma_a \phi V)_{fuel}}{\sum_i (\Sigma_a \phi V)_i} - \dfrac{(\Sigma_a \phi V)_{fuel}}{\sum_i (\Sigma_a \phi V)_i + (\Sigma_a \phi V)_{poison}}}{\dfrac{(\Sigma_a \phi V)_{fuel}}{\sum_i (\Sigma_a \phi V)_i + (\Sigma_a \phi V)_{poison}}}$$

$$= \frac{(\Sigma_a V)_{poison}}{\sum_i (\Sigma_a V)_i}$$

The fluxes in these terms cancel out because the medium is completely homogeneous.

If the medium is homogeneous but the poison is of a heterogeneous nature, the same formalism may be kept with the introduction of the disadvantage factor for the poison which is defined as,[1]

$$F = \frac{\phi_s}{\overline{\phi}_{poison}}$$

where ϕ_s refers to the flux at the surface of the poison and $\overline{\phi}$ refers to the average flux in the poison.

Then

$$\Delta k_{\infty} = \frac{\frac{\Sigma_a V}{F}\ \text{poison}}{\sum_i (\Sigma_a V)_i} \qquad (1)$$

The quantity of poison necessary to reduce k_{∞} of a test medium to unity may be found in a finite reactor by providing a cavity in the reactor into which a sample of the poisoned medium may be placed and by providing means for adjusting the neutron energy spectrum incident on this cavity such that the incident spectrum is identical to that which would exist in an infinite medium of the test material.

The above is a simple description of the Physical Constants Testing Reactor at Hanford Atomic Products Operation, Richland, Washington. A cavity is provided which is much larger than the test sample so that a layer of the medium, the buffer region, surrounds the test sample and brings the neutron energy spectrum into equilibrium. Further adjustment of the spectrum is made by changing the loading configuration of the reactor external to the cavity.

Equation (1) refers to the condition in which the test sample has been correctly poisoned. In actual practice this condition is not fulfilled in that it is quite difficult to obtain the exact amount of poison necessary. Instead the test sample is poisoned quite close to the correct amount and then a small amount of poison is added and an extrapolation is made to the correct amount. This extrapolation is made as follows:

By methods of perturbation theory it can be shown that[4]

$$\Delta\rho = \alpha M_{\text{poison}}\ \phi_{1/v}$$

where M_{poison} = mass of poison; α = proportionality constant

$$\Delta\rho = \rho_{\text{poison}} - \rho_{\text{void}}$$

and ρ is defined as the reactivity of the reactor.

Then rewriting equation (1)

$$\Delta k_{\infty} = \frac{\frac{(\Sigma_a V)}{F}\text{poison} + \left(\frac{(\Sigma_a V)}{F}\text{poison}'\right)\gamma}{\sum_i (\Sigma_a V)_i}$$

where

$$\gamma = \left(\frac{\Delta\rho}{\phi_{1/v}}\right)_{\text{void}} \left(\frac{\phi_{1/v}}{\Delta\rho}\right)_{\text{poison}'}$$

$$= \left(\frac{\rho_{\text{poison}} - \rho_{\text{void}}}{\rho_{\text{poison}} - \rho_{\text{poison}'}}\right) \frac{\phi_{1/v\ \text{poison}'}}{\phi_{1/v\ \text{void}}}$$

This may be rewritten as

$$\Delta k_{\infty} = \frac{\left(\frac{N\sigma_a}{F}\right)\text{poison} + \left(\frac{N\sigma_a}{F}\right)\text{poison}'\left(\frac{\rho - \rho_{\text{void}}}{\rho - \rho'}\right)\left(\frac{\phi_{1/v'}}{\phi_{1/v\ (\text{void})}}\right)}{\sum_i (N\sigma_a)_i}$$

where N_i = total number of gram atoms or molecules of the particular material

$(\sigma_a)_i$ = microscopie absorption cross section of the particular material

Details

The material, 2 w/o U^{235} enriched UF_4 paraffin moderated at a hydrogen-to-U^{235} atomic ratio of 195, was fabricated at the Oak Ridge National Laboratory, Oak Ridge, Tennessee.[2] The composition of the material was 92.0 per cent UF_4 and 8 per cent paraffin by weight. This gave a uranium concentration of 0.698 grams uranium per gram of material. The average particle size of the UF_4 was less than 0.020 inch. The material was pressed into blocks of various sizes (4 inches x 4 inches x 4 inches, 1 inch x 4 inches x 4 inches, 2 inches x 2 inches x 4 inches, etc.)

for ease of handling and assembly. Each of these blocks was wrapped in a thin piece of aluminum foil to eliminate any contamination problems in handling the material.

This particular material was chosen because of a need for information in this general range of U^{235} enrichment, and because critical experiments could easily be conducted and thus a comparison of the values of the infinite medium thermal neutron multiplication factor obtained by two different methods could be made.

The poison used in this experiment consisted of boron carbide impregnated polyethylene containing 4 w/o boron carbide.[7] This material was in the form of 0.005 inch thick blown tubing. It was developed in the past for use in experiments of this type. The neutron absorption cross section of this poison was measured with respect to standard copper by danger coefficient methods in the Physical Constants Testing Reactor.

The assembled system was 24 inches x 24 inches x 30 inches with a 6-inch x 6-inch x 12-inch central section used as the test sample. This gave a 9-inch buffer region surrounding the test sample.

A 6-1/2-inch x 6-1/2-inch x 30-1/2-inch rectangular tube constructed from 1/4-inch 61ST aluminum was used to house the test sample and the buffer material on either end of the test sample. The test sample and two end buffers were contained in rectangular tanks. These tanks were constructed of 1/8-inch 61ST aluminum. The tank for the test sample was 6-1/4 inches x 6-1/4 inches x 12-1/4 inches and the two end buffer tanks were 6-1/4 inches x 6-1/4 inches x 9-1/4 inches. Figure 1 shows the partially assembled system and some of the different size blocks and Figure 2 shows the assembled system with the tanks being put in place.

A total of 1321.14 kg of 2 w/o U^{235} enriched UF_4 paraffin moderated at a hydrogen-to-U^{235} atomic ratio of 195 were used. This assembly contained 18.4 kg of U^{235}.

Previous to this experiment the bare critical size of this material had been measured by Oak Ridge National Laboratory and was found to contain 25.8 kg U^{235}.[2] For a graphite reflected system the minimum critical mass was considerably less, and a good deal more than the minimum critical mass would be present when the system was assembled in the cavity of the Physical Constants Testing Reactor which is graphite moderated. For this reason, extreme care had to be exercised while assembling the system. Each individual block was wrapped with poison before being placed in the cavity. The amount of poison had been previously determined from theoretical calculations of k_∞ and from an estimate of k_∞ based on the work done at Oak Ridge National Laboratory. The theoretical calculation of k_∞ is shown in the Appendix. Based on these predictions of k_∞ the assembled poisoned system would have k_∞ equal to unity. Tables I and II give the mass of each material used in the test sample and buffer region as well as the 2200 meter/sec cross section values and atomic or molecular weights.

Some copper was used to poison down the buffer region in addition to the boron carbide impregnated polyethylene. The reasion for using copper was that since the boron carbide impregnated polyethylene was still in somewhat of a developmental stage not enough of it was available to do the complete poisoning.

The test sample tank was provided with holes and the blocks of material in it were drilled so that flux traverse measurements could be made in it. These holes were in the longitudinal and transverse directions to the test sample and extended three inches into the buffer region so that the traverses could be extended out into the buffer region.

The flux traverses were made by irradiating bare and cadmium covered gold foils placed in positions along the traverse holes. Figure 3 shows the traverse bars and the gold foils in place. These foils are 0.25 inch x 0.5 inch x 0.005 inch gold and the cadmium dishes used were 0.040 inch thick. The activity of the foils was obtained by placing them on a 3-inch crystal scintillation counter and reading the gamma from

Au^{198} decay. The activities were normalized to unit mass of the foils and to a constant reactor power level by use of a monitor foil placed outside the assembly.

A test sample tank filled with helium was used as a substitute for a void. Helium is very good for this purpose because of its low neutron absorption and scattering cross sections.

The method used to find the correct amount of poison and hence k_∞ was then to measure the reactivity of the reactor at a particular control rod setting with the test sample in place. The test sample was then removed, and with the control rods positioned at the same previous setting, a reactivity measurement of the test sample tank filled with helium was made. The poison in the test sample was then adjusted towards the correct value. A flux traverse measurement was then made and the poison in the buffer region adjusted to obtain the correct incident flux. These measurements were repeated and adjustments made until the test sample was poisoned to within one per cent of the correct value and the correct neutron energy spectrum was incident on the test sample. A small amount of poison was then added to the test sample and the reactivity and flux traverse measurements were again made. Then as shown in previous arguments an extrapolation was then made to the correct value.

Further measurements were made to observe the effect of over-poisoning the buffer and poisoning the test sample completely heterogeneously. In the latter experiment the poison was placed outside the test sample rather than around the individual blocks. The purpose of these experiments was to varify some work done previously on low U^{235} enrichments for which the heterogeneous type poisoning was used.[8] Perturbation theory would predict that each of the two methods give the correct answer.

Figures 4 and 5 are examples of the traverses made with the bare and cadmium covered gold foils. The cadmium ratio plots were used to insure that the incident neutron energy spectrum was the correct spectrum for the material.

Table III shows the values of k_∞ and other results obtained from experimental data before correction factors were introduced.

It can be shown by two-group analysis that the error in $k_\infty - 1$ can be minimized by requiring that the cadmium ratio be constant in the test sample and out into the buffer region.[4] The error involved here results from spectral mismatching in the test sample. An expression for this error may be written

$$\sigma(\Delta k_\infty) = \frac{\left(\frac{\phi_1'}{\phi_2'} - \frac{\phi_1}{\phi_2}\right)\left(\frac{m_1'}{m_2'} - \frac{m_1}{m_2}\right)}{\left(\frac{\phi_1}{\phi_2}\right)\left(\frac{m_1}{m_2}\right)}$$

where ϕ_1/ϕ_2 and m_1/m_2 are the fast-to-slow flux and adjoint flux ratios which would be found in the infinite critically poisoned medium and ϕ_1'/ϕ_2' and m_1'/m_2' are the similar flux ratios in the test sample. Thus if either the flux spectrum or the adjoint flux spectrum is matched, the spectral error in k_∞ will vanish.

By varying the cadmium ratio over the range involved, it was determined that the error due to a possible mismatch in the spectrum was no greater than three-fourths of one per cent in Δk_∞. This is included as an error in the error analysis.

The assumption has been made that the addition of poison to the system changes only the thermal utilization of the system. The addition of the poison has two effects. First, it causes a shift in the neutron temperature, and second, because of the polyethylene contained in the poison the hydrogen-to-U^{235} ratio was slightly increased. The second of these was the largest and can be corrected by observing the effect of the addition of a small amount of polyethylene and extrapolating this effect back to the correct value. The first effect was more difficult to determine; it results from changes in η which were dependent upon neutron temperature.

One of the most difficult problems in an experiment of this type was the determination of the proper absorption cross-section values for the different materials. Because of the low moderation and high enrichment the effective neutron temperature was considerably higher than for most common thermal reactor systems. The use of standard 2200 meter/sec cross-section values for the different materials in the system might be introducing an error because of the higher neutron temperature and in addition the neutron energy spectrum was not the common Maxwellian with 1/E tail for which these cross sections were designed.[10] The magnitude of this effect can be determined by use of some fairly recent developments in this field. A table of effective cross-section values has been tabulated for systems containing a mixture of hydrogen, U^{235} and a 1/v absorber.[9] These cross sections were obtained by calculating the Wigner-Wilkens spectrum for different mixtures of these ingredients and averaging the cross sections over the spectrum by dividing the spectrum into 30 groups. All the materials in this experiment except U^{235} and hydrogen were then classified as the 1/v absorber. Any non-1/v effect of carbon and fluorine are negligible due to their low cross sections and boron is strictly a 1/v absorber. Then the only discrepancy in this method is the effect the U^{238} resonances have on the spectrum.

The effect of any change in η can also be accounted for using these effective cross-section values. The analysis is as follows:

$$\frac{k_\infty - k'_\infty}{k'_\infty} = \frac{\eta \epsilon p f - \eta' \epsilon' f'}{\eta' \epsilon' p' f'} = \Delta k_\infty$$

$$\epsilon = \epsilon' \qquad p = p'$$

therefore

$$\Delta k_\infty = \frac{\eta f - \eta' f'}{\eta' f'}$$

When this method was used a value of k_{∞} = 1. 216± 0. 013 was obtained. The error in this measurement was the result of two conditions. The first was errors due to experimental procedure and measured errors in 2200 meter/sec cross-section values. This amounted to ± 0. 005. The second was due to errors in the "effective" cross-section values which were used. Only an estimate of this can be made. The error was chosen as the discrepancy between the value of k_{∞} obtained using 2200 meter/sec cross-section values and the value obtained using the "effective" cross-section values. This amounted to ± 0. 008.

The critical experiments at Oak Ridge National Laboratory gave a value of k_{∞} = 1. 200 ± 0. 011 when a one-group model was used in the analysis and a value of 1. 202 ± 0. 012 when a two-group model was used.[12] This constitutes good agreement between the PCTR experiments and the Oak Ridge critical experiments.

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APPENDIX

THEORETICAL CALCULATION OF k_∞

The value of k_∞ may be calculated by the use of standard 2200 meter/sec cross-section values and a knowledge of the materials in question. The method used was to calculate k_∞ from the four factor formula

$$k_\infty = \eta \epsilon p f$$

where ϵ, η, p, and f have their usual meaning and are written as follows:

$$\eta = \frac{\nu N^{235} \sigma_f^{235}}{N^{235} \sigma_a^{235} + N^{238} \sigma_a^{238}}$$

$$p = e^{-\frac{3.9}{\bar{\xi}}} \left[\frac{N_o^{238}}{\Sigma_s} \right]^{.585}$$

$$f = \frac{N^{235} \sigma_a^{235} + N^{238} \sigma_a^{238}}{N^{235} \sigma_a^{235} + N^{238} \sigma_a^{238} + N^{C_{25}H_{52}} \sigma_a^{C_{25}H_{52}} + N^F \sigma_a^F}$$

where

$\nu = 2.47$ $\qquad$ $\sigma_f^{235} = 571$ b

N^{235}	= .0203	σ_a^{235}	= 681	σ_s^{235}	= 10	ξ^{235}	= .00838
N^{238}	= .9797	σ_a^{238}	= 2.71	σ_s^{238}	= 8.3	ξ^{238}	= .00838
N^F	= 4	σ_a^F	= .010	σ_s^F	= 3.9	ξ^F	= .1000
N^H	= 3.939	σ_a^H	= .332	σ_s^H	= 20	ξ^H	= 1.00
N^C	= 1.893	σ_a^C	= .0034	σ_s^C	= 4.8	ξ^C	= .158

$$\bar{\xi} = \frac{\sum_i \left(\xi_i N_i \sigma_{si} \right)}{\sum_i \left(N_i \sigma_{si} \right)}$$

$$\Sigma_s = \sum_i \left(N_i \sigma_{si} \right)$$

The calculation of ϵ is taken from a partly theoretical, partly empirical formula whose only justification is that from past experience it seems to work.

Then

$$\epsilon = 1 + \left(\frac{\nu \Sigma_f^{238}}{\Sigma_T} - \frac{\Sigma_a^{238}}{\Sigma_T} \right) \left[\left(\frac{\Sigma_e + \nu \Sigma_f^{238}}{\Sigma_T} \right) (\overline{\gamma} - 1) + 1 \right]$$

ν = 2.55 neutrons per fission

Σ_f = .29 cm^{-1}

Σ_a = .39 cm^{-1}

$\Sigma_e = \sum_i (\alpha \sigma_e)_i$ = 36.24 cm^{-1}

α = number of atoms per cm^3

σ_e = microscopic elastic scattering cross section

$\sum_i$ = sum on all materials

$\Sigma_T = \Sigma_e + \Sigma_f^{238} + \Sigma_I^{238} + \Sigma_a^{238}$ = 39.04 cm^{-1}

Σ_I = macroscopic inelastic scattering cross section of U^{238}

$$\overline{\gamma} = \frac{\sum_i (\gamma \alpha \sigma_e)_i}{\sum_i (\sigma_e \alpha)_i}$$

$$\gamma_i = \frac{\ln \frac{E_o}{E_1}}{\xi_i}$$

E_o = 2 mev

E_1 = 1.1 mev (U^{238} fission threshold)

$\overline{\gamma}$ was found to be 1.67 collisions.

$\epsilon - 1$ has to be reduced by approximately 3 per cent to account for the inadequate treatment of the energies of fission neutrons. Then from the above formulas the following values were obtained:

p = .747

ηf = 1.60

ϵ = 1.03

Then

$$k_{\infty} = 1.23$$

The error in a calculation of this type has normally been assumed to be about 5 per cent of k_{∞}. The error was mainly due to inadequate theoretical treatment and errors in cross section values.

Then the value of k_{∞} from a theoretical calculation was in good agreement with the experimental value of k_{∞} for this material.

COMPARISON OF RESULTS FROM PCTR k_{∞} MEASUREMENT AND ORNL CRITICALITY EXPERIMENTS

The bare critical size of a reactor may be obtained from the critical buckling of the reactor. The critical buckling may be obtained from the following equation if values for k_{∞}, L^2 and τ are known.

$$1 = \frac{k_{\infty} e^{-B^2 \tau}}{1 + L^2 B^2}$$

Where B^2 is the geometrical buckling, L^2 is diffusion length squared, and τ is the age to thermal, and k_{∞} is the experimental value (1.216).

The following formulas were used to obtain these quantities:[1, 11]

$$L^2 (H/U = 195) = L^2_{(H_2O)} \frac{(\Sigma_{tr} \Sigma_a)_{(H_2O)}}{(\Sigma_{tr} \Sigma_a)_{(H/U = 195)}}$$

$$\tau(H/U = 195) = \frac{\tau(\text{paraffin})}{\left(\frac{\rho(\text{mod.})}{\rho(\text{paraffin})} + \frac{\Sigma_{sl}^{\text{fuel}}}{\Sigma_{sl}^{\text{mod.}}}\right)\left(\frac{\rho(\text{mod.})}{\rho(\text{paraffin})} + \frac{\Sigma_{tr}^{\text{fuel}}}{\Sigma_{tr}^{\text{mod.}}}\right)}$$

$$\tau_{(paraffin)} = \tau_{(H_2O)} \frac{(\Sigma_s \Sigma_{tr})\,(H_2O)}{(\Sigma_s \Sigma_{tr})_{(paraffin)}}$$

$\Sigma_{sl} = \Sigma_s + \Sigma_{inelastic}$

$\Sigma_{tr} = \Sigma_s (1 - \overline{\mu}_o)$ $\quad \overline{\mu}_o = \frac{2}{3A}$ (A = atomic or molecular weight)

$\rho_{(mod.)}$ = density of moderator

$\rho_{(paraffin)}$ = density of paraffin

$$B^2 = \left(\frac{2.405}{R + \lambda}\right)^2 + \left(\frac{\pi}{H + 2\lambda}\right)^2$$

λ = 3.4 cm = bare extrapolation length

$L^2_{(H_2O)} = 7.45\ cm^2$

$\tau_{(H_2O)} = 27$ to $31.4\ cm^2$ (Without actual measurements of τ it is difficult to establish a value better than these limits.)

When the above formulas were applied, the amount of U^{235} contained in the critical mass of a bare "square" cylinder was found to be from 23.6 to 30.1 kg. U^{235} depending on the value of $\tau_{(H_2O)}$ (27 to 31.4 cm^2). The value obtained from critical experiments performed at the Oak Ridge National Laboratory was approximately 28 kg U^{235}.(2) Again there is good agreement between the two methods within the restrictions imposed by the $\tau_{(H_2O)}$ values.

ERROR ANALYSIS

The error in the measurement of k_∞ is divided into three main parts.

1. Error due to spectral mismatch and possible changes in η and p due to the addition of poison.
2. Error due to the use of incorrect cross section for these materials caused by non-Maxwellian 1/v type spectra and higher effective neutron temperatures.

3. Errors in the standard cross section values and in experimental methods.

The first two have been discussed in the main part of the text and the third will be discussed here.

The method of determining this error was to use standard propagation of error technique.

$$\sigma^2(T) = \sum_i \left[\frac{\partial T}{\partial x_i} \sigma(x_i) \right]^2$$

Then from the equation

$$\Delta k_\infty = \frac{(\Sigma_a V)_{poison}}{\sum_i (\Sigma_a V)_i}$$

where $(\Sigma_a V)_{poison}$ is the correct poison.

One can write

$$\Delta k_\infty = \frac{\frac{(\Sigma_a V)}{F}_{poison} + \frac{(\Sigma_a V)'}{F}_{poison} \left(\frac{\rho_1 - \rho_{He}}{\rho_1 - \rho_2} \right) \frac{\phi_{1/v2}}{\phi_{1/v He}}}{\sum_i (\Sigma_a V)_i}$$

and further

$$\Delta k_\infty = \frac{M \frac{\sigma_a}{FW}_{poison} + M \frac{\sigma_a'}{FW}_{poison} \left(\frac{\rho_1 - \rho_{He}}{\rho_1 - \rho_2} \right) \left(\frac{\phi_{1/v\,2}}{\phi_{1/v He}} \right)}{M \frac{\sigma_a}{W}_{U^{238}} + M \frac{\sigma_{a\,f}}{W}_{U^{235}} + M \frac{\sigma_a}{W}_{C_{25}H_{52}} + M \frac{\sigma_a}{W}_{F_{19}}}$$

where

M = mass of material in test sample

W = atomic or molecular weight

Let

$$a = \left(M \frac{\sigma_a}{WF}\right)^2$$

$$b = \left(M \frac{\sigma_a'}{WF}\right)^2 \left(\frac{\rho_1 - \rho_{He}}{\rho_1 - \rho_2}\right)^2 \left(\frac{\phi_{1/v2}}{\phi_{1/vH2}}\right)^2$$

$$c = \left[\frac{1}{\left(M\frac{f\sigma_a}{W}\right)_{U^{235}} + \left(M \frac{\sigma_a}{W}\right)_{238} + \left(M \frac{\sigma_a}{W}\right)_{C_{25}H_{52}} + \left(M \frac{\sigma_a}{W}\right)_{CH_2} + \left(M\frac{\sigma_a}{W}\right)_{F-19}}\right]^2$$

$$d = \left(M \frac{\sigma_a f}{W}\right)^2_{U^{235}}$$

$$e = \left(M \frac{\sigma_a}{W}\right)^2_{U^{238}}$$

$$f = \left(M \frac{\sigma_a}{W}\right)^2_{C_{25}H_{52}}$$

$$g = \left(M \frac{\sigma_a}{w}\right)^2_{CH}$$

$$h = \left(M \frac{\sigma_a}{W}\right)^2_{F-19}$$

then

$$\sigma(\Delta k_\infty)^2 = C\,a \left[\frac{\sigma(M)^2_{poison}}{M^2_{poison}} + \frac{\sigma\left(\frac{\sigma}{w}\right)^2_{poison}}{\left(\frac{\sigma}{w}\right)^2_{poison}} + \frac{\sigma(F)^2_{poison}}{F^2_{poison}}\right]$$

$$+ C\,b \left[\frac{\sigma(M')^2_{poison}}{M^2_{poison}} + \frac{\sigma\left(\frac{\sigma}{w}\right)^2_{poison}}{\left(\frac{\sigma}{w}\right)^2_{poison}} + \frac{\sigma(F)^2_{poison}}{F^2_{poison}}\right.$$

$$\left. + \frac{\sigma(\rho_1)^2 + \sigma(\rho_{He})^2}{(\rho_1 - \rho_{He})^2}\right.$$

$$+ \frac{\sigma(\rho_1)^2 + \sigma(\rho_2)^2}{(\rho_1 - \rho_2)^2} + \frac{\sigma(\phi_{1/v2})^2}{\phi_{1/v_2}^2} + \frac{\sigma(\phi_{1/vHe})^2}{\phi_{1/v\ He}^2}\Bigg]$$

$$+ C^2(\sqrt{a} + \sqrt{b})^2\Bigg[d\left(\frac{\sigma(M_{235})^2}{M_{235}^2} + \frac{\sigma\left(\frac{\sigma}{w}\right)_{235}^2}{\left(\frac{\sigma_a}{w}\right)_{235}^2} + \frac{\sigma(f_{235})^2}{f_{235}^2}\right)$$

$$E\left(\frac{\sigma(M_{238})^2}{M_{238}^2} + \frac{\sigma\left(\frac{\sigma}{w}\right)_{238}^2}{\left(\frac{\sigma_a}{w}\right)_{238}^2}\right) + f\left(\frac{\sigma\left(M_{C_{25}H_{52}}\right)^2}{M_{C_{25}H_{52}}^2} + \frac{\sigma\left(\frac{\sigma}{w}\right)_{C_{25}H_{52}}^2}{\left(\frac{\sigma_a}{w}\right)_{C_{25}H_{52}}}\right)$$

$$G\left(\frac{\sigma\left(M_{CH_2}\right)^2}{M_{CH_2}^2} + \frac{\sigma\left(\frac{\sigma}{w}\right)_{CH_2}^2}{\left(\frac{\sigma_a}{w}\right)_{CH_2}^2}\right) + h\left(\frac{\sigma(M_{F-19})^2}{M_{F-19}^2} + \frac{\sigma^2\left(\frac{\sigma}{w}\right)_{F-19}}{\left(\frac{\sigma_a}{w}\right)_{F-19}^2}\right)\Bigg]$$

Values of $\frac{\sigma(M_i)}{M_i}$ and $\frac{\sigma\left(\frac{\sigma}{w}\right)_i}{\frac{\sigma}{w}_i}$ have been given in Table I.

Other values were

$$\frac{\sigma(\rho_i)^2 + \sigma(\rho_2)^2}{(\rho_1 - \rho_2)^2} = 1.51 \times 10^{-2} \qquad \frac{\sigma(\rho_1)^2 + \sigma(\rho_{He})^2}{(\rho_1 - \rho_{He})^2} = 1.194 \times 10^{-2}$$

$$\frac{\sigma(\phi_{1/vi})^2}{\phi_{1/v\ i}^2} = 4\ 66 \times 10^{-3}$$

Then

$$\sigma(k_\infty)^2 = 13\ 44 \times 10^{-6}$$

Then the error introduced because of incorrect 2200 meter/sec cross sections and because of experimental methods was found to be

$$\sigma(\Delta k_\infty) = 3.7 \times 10^{-3}$$

TABLE I

MATERIALS IN TEST SAMPLE

Material	Weight (g)	Molecular Weight	σ_a (at 2200 meters/sec) in barns	(σ_a/w)* W = atomic or molecular wt
U^{238}	21562 ± 62	238.125	2.71	.01138 ± .00008
U^{235}	440 ± 4	235.117	681	2.896 ± .036
Paraffin $C_{25}H_{52}$	2522 ± 9	352.796	17.349	.04918 ± .00030
Fluorine F-19	6998 ± 25	19.005	<.010	<.0005
Polyethylene CH_2	167.1 ± .4	14.031	.6674	.04756 ± .00028
Poison (4 wt % Boron Carbide Impregnated Polyethylene)	174.0 ± .4	--	--	2.086 ± .022 **

* All erros are standard deviations.

** Measured value.[3] All other values of σ_a (2200 meters/sec) taken from BNL-325, Second Edition 1958 (see reference 10).

TABLE II

MATERIALS IN BUFFER REGION

Material	Weight in kg
U^{238}	876.67
U^{235}	17.90
Paraffin $C_{25}H_{52}$	102.60
Fluorine F-19	284.66
Polyethylene CH_2	3.22
Poison	
(1) Borated Polyethylene	3.36
(2) Copper	102.60

Because of the lack of the boron carbide impregnated polyethylene some copper had to be used in poisoning down the buffer region. To avoid any possible complications the copper was placed toward the outside and the boron impregnated polyethylene placed toward the central region.

TABLE III

EXPERIMENTAL DATA AND VALUES OBTAINED

Type of Incident Flux *	$\frac{\rho_1 - \rho_{He}}{\rho_1 - \rho_2}$	$\frac{\phi_{1/v_2}}{\phi_{1/v_o}}$ **
Thermal Loading	1.126	1
Intermediate Loading	1.041	1
Fast Loading	1.196	1
Intermediate Loading with addition of 5959 g of Cu on front face	1.293	1
Intermediate Loading with addition of 5959 g of Cu on front face plus 19.3 g poison on inside of rectangular tube at test cell	1.344	1
Intermediate Loading with poison placed on outside of test sample instead of around the individual blocks	1.506	1

* These descriptions of the incident flux are only relative to one another.

** The actual experimental values of $\phi_{1/v_2} / \phi_{1/v_o}$ were not exactly unity; however, they are unity within the experimental error.

FIGURE 1

Partially Assembled System and Some of the Block Sizes

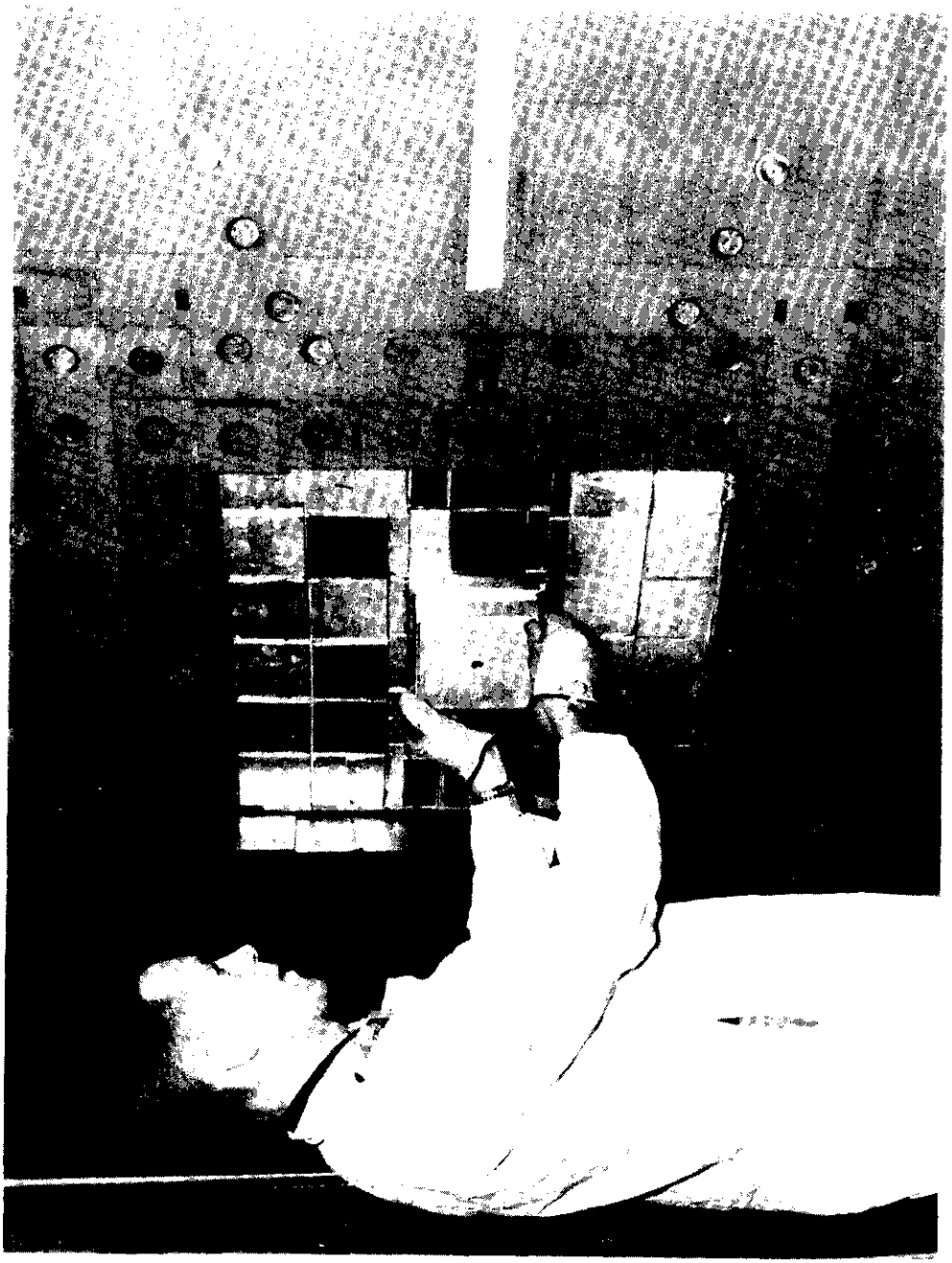

FIGURE 2
Insertion of Test Sample into Assembled System

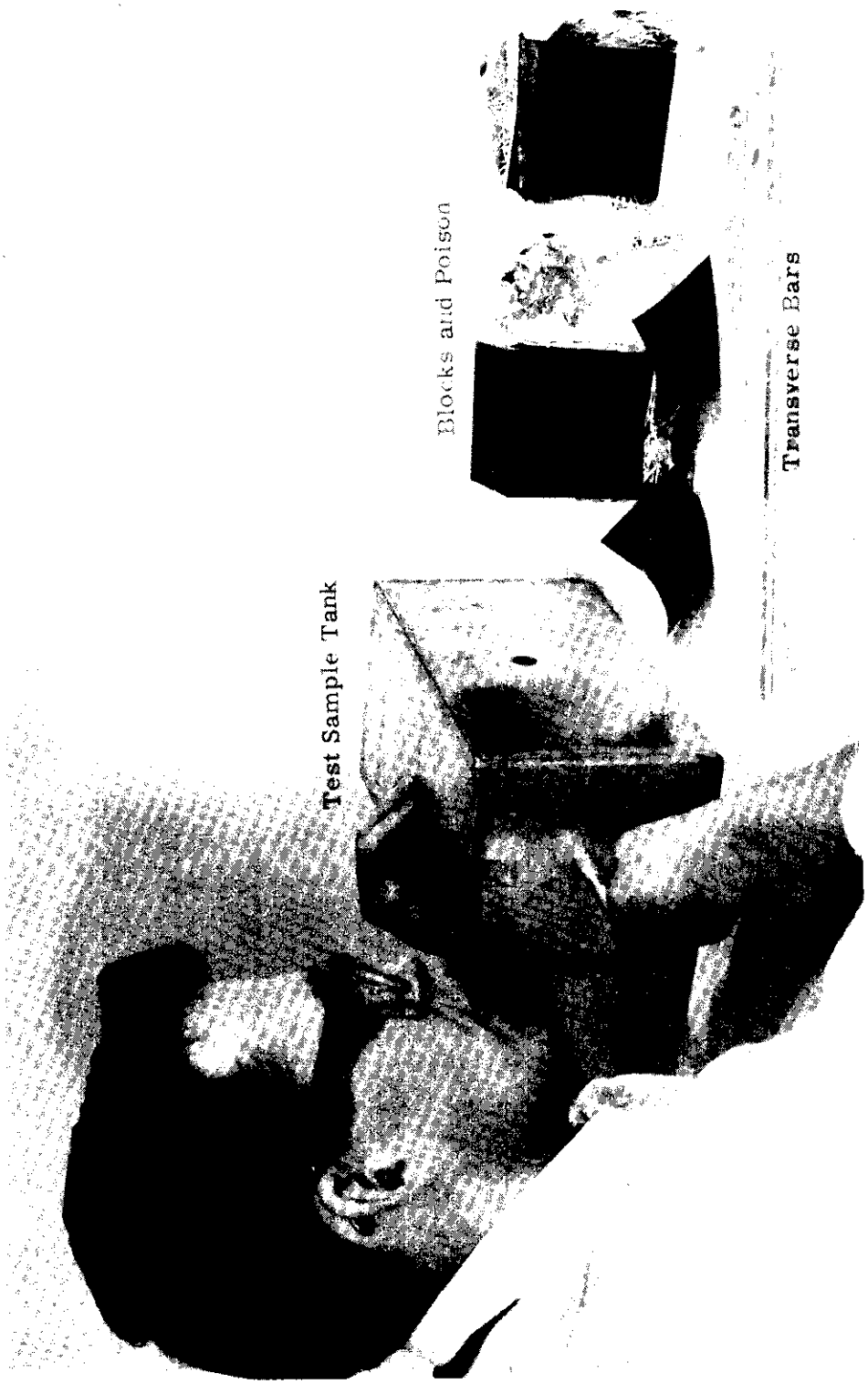

FIGURE 3
Test Sample and Traverse Materials

AEC-GE RICHLAND, WASH.

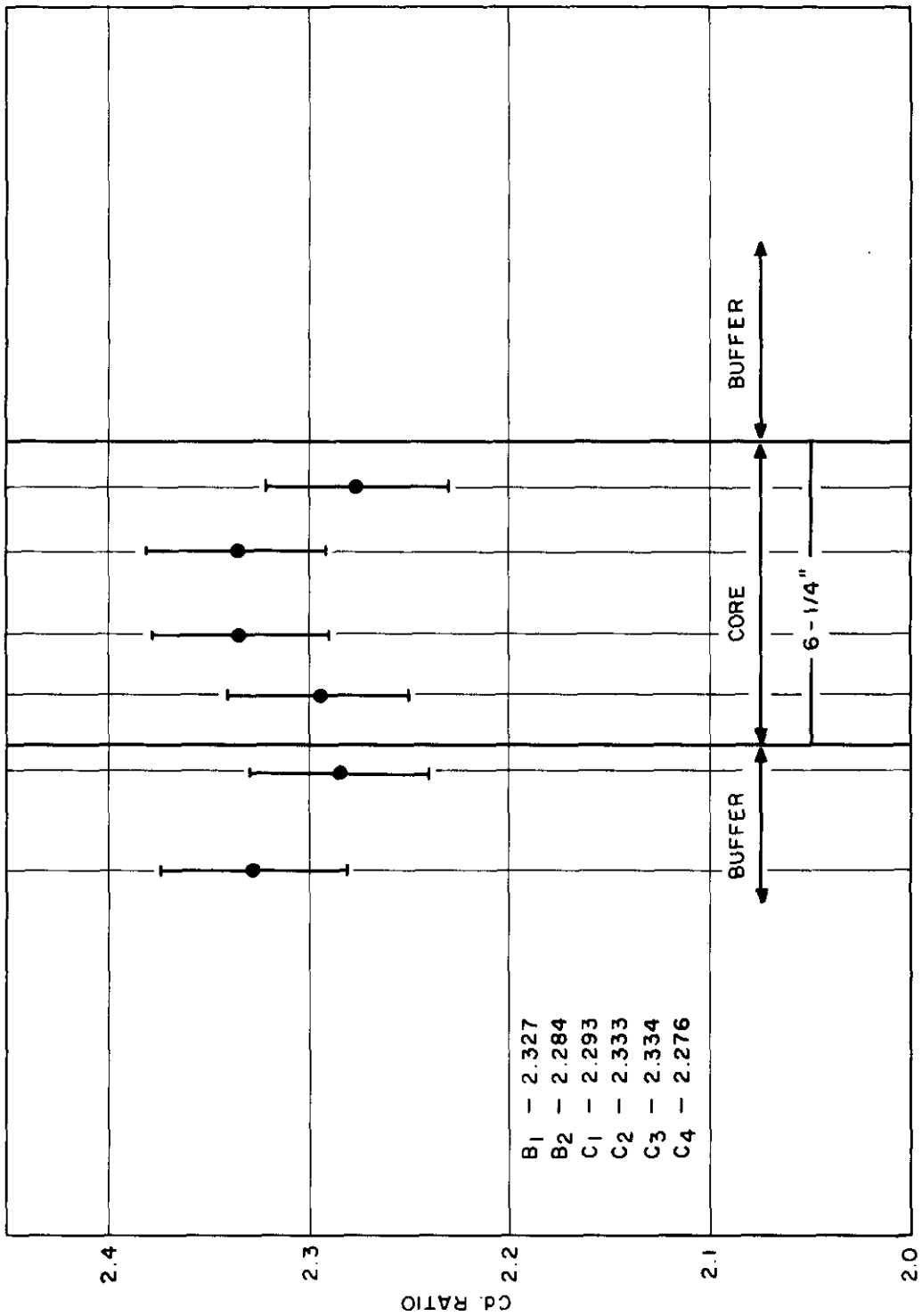

FIGURE 4

Radial Cd. Ratios vs Foil Position

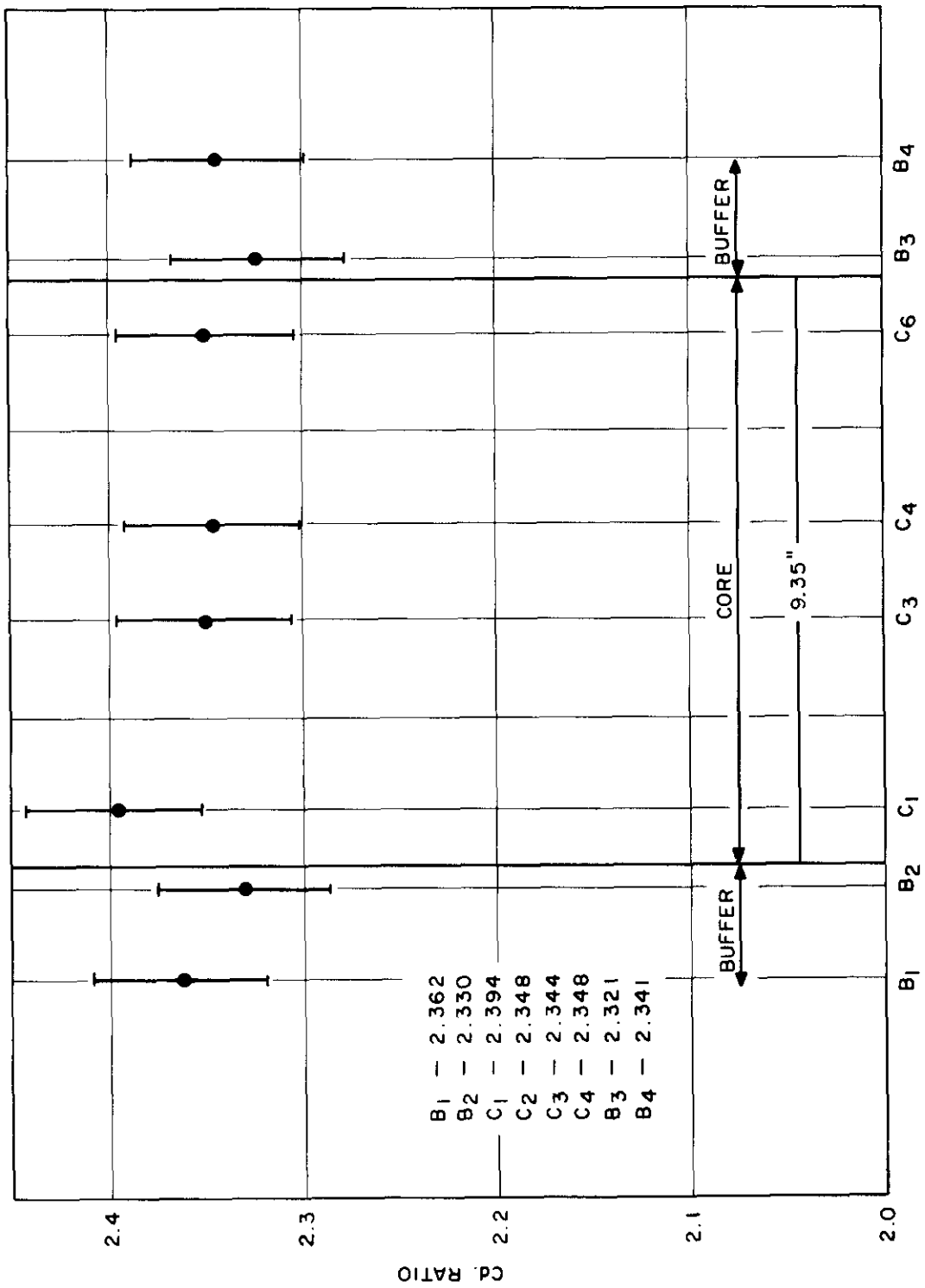

FIGURE 5

Longitudinal Cd. Ratios vs Foil Position

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