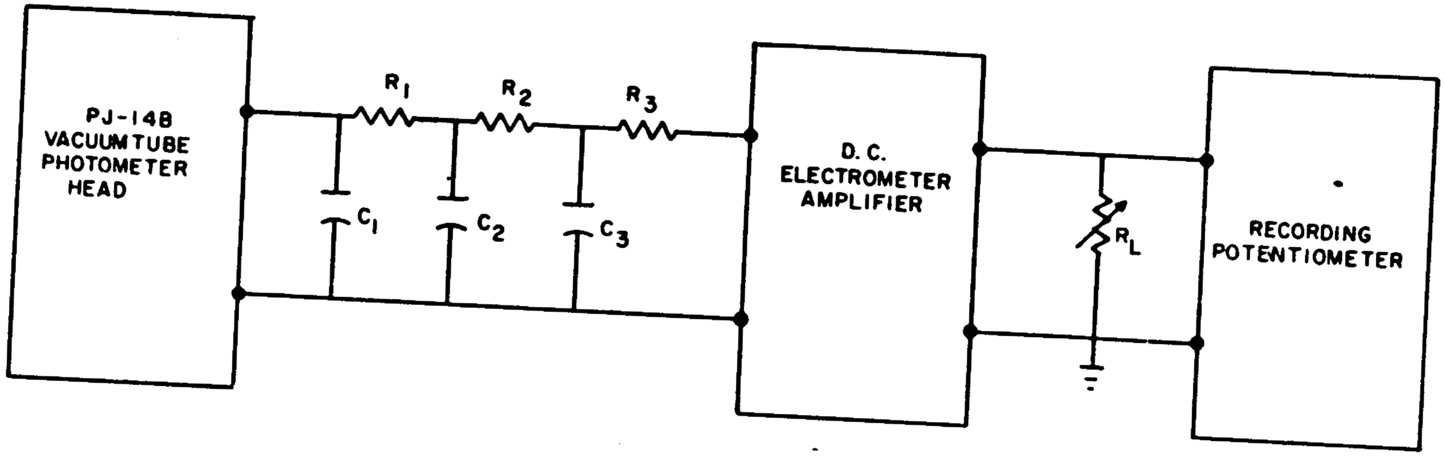



$C_1$, $C_2$, $C_3$: 0.05 mfd., 1200 volt Mica

$R_1$, $R_2$, $R_3$: 2.2 megohms

$R_L$: Calibration Adjusting Resistor, Decade Box 9X (10+1+0.1+0.01)

Figure 12 PJ-14B PHOTOTUBE PHOTOMETER- The resistance-capacitance network between the photometer head and the amplifier is used in the photometry of flashing lights and is otherwise removed.

# 3. CALIBRATION OF THE PHOTOMETRIC SYSTEM

## 3.1. Introduction

Lamp standards of luminous intensity are used for calibrating the photometric testing equipment; a separate calibration is made before each test, and a record is kept of the photometer sensitivity in order to detect any irregularities. The illumination on the photosensor produced by the standard lamp is adjusted to some typical value of the illumination produced by the test light, usually in the range of 75% to 100% of the peak illumination produced by the test light. This procedure minimizes errors resulting from nonlinearity of the response of the photosensor. The adjustment of the illumination of the standard lamp on the photosensor is accomplished by varying the distance of the standard lamp from the photocell and by using optical attenuators. The photometer is usually calibrated so that it is direct reading in luminous intensity.

## 3.2. Standard Lamps

The standard lamps used are "working standards" whose luminous intensity in a given direction has been determined at a given voltage.[10] Standard lamps are available ranging in intensity from about 8 to 900 candles. When a colored light is being tested, a filter is placed between the standard lamp and the photosensor, which results in a standard lamp-filter combination having approximately the same spectral characteristics as those of the light to be tested. This procedure minimizes errors resulting from inadequate spectral correction of the photosensor. In this procedure, a standard lamp of known color temperature as well as of known luminous intensity is needed.

## 3.3. Attenuators

Sector disks are almost always used for light attenuation, although neutral filters are also available. A sector disk is usually placed between the standard lamp and the photosensor to calibrate the photometer for the proper range of illumination. However, when the intensity of the light being photometrically measured is unusually high, the sector disk may be used to attenuate the illumination from the test light. The range of sector disks available is from 1% to 80% transmittance.

When a sector disk is used, it is placed within a few inches of the photocell in order to reduce error from stray light. The disk is rotated at a few hundred revolutions per minute, which is fast enough to minimize error from apparent flicker. When a high illumination is attenuated by a sector disk of low transmittance, there is an error which results from only one part of the photocell being illuminated at a time; this error is successfully eliminated by placing a condenser of about 4 mfd. across the output of the photocell. (In utilizing this technique, one must be careful to obtain a capacitor which does not itself generate an emf.)

### 3.4. Calibration Procedure

The calibration involves illuminating the photosensor with light from a standard lamp placed at a given distance from the photosensor, and then adjusting the sensitivity of the photometric system to some desired value.

If $\underline{i}$ is the photosensor current,
$\underline{I}$ is the intensity of the light illuminating the photosensor,
and $\underline{D}$ is the distance from the test unit to the photosensor,
then, since the photosensor produces a current proportional to the illuminance on its face,

$$i = kI/D^2 \tag{1}$$

where $\underline{k}$ is the sensitivity of the photosensor.

It is usually convenient to calibrate the photometer to be direct reading, so that

$$I = N\delta \tag{2}$$

where $\underline{\delta}$ is the reading of the potentiometer of the measuring circuit, and where $\underline{N}$ is an integral power of 10 or the product of an integer, usually 2 or 5, and an integral power of 10.

The photometer is then calibrated by using a standard lamp of known horizontal luminous intensity. If

$\underline{I_s}$ is the luminous intensity of the standard lamp,
$\underline{D_s}$ is the distance of the standard lamp from the photosensor,
$\underline{\delta_s}$ is the potentiometer reading and $\underline{i_s}$ is the photosensor

current when the photosensor is illuminated by light from the standard lamp placed at the distance $\underline{D_s}$ from the photosensor, then

$$i_s = k\, I_s/D_s^{\,2} \tag{3}$$

and since the potentiometer reading is proportional to the photosensor current,

$$i_s/\delta_s = i/\delta \tag{4}$$

Combining (1), (2), (3), and (4),

$$\delta_s = I_s D^2/ND_s^{\,2} \tag{5}$$

Calibration is accomplished by the following procedure: $\underline{I_s}$ and $\underline{D_s}$ are chosen so that $I_s/D_s^2$ will be approximately equal to $I/D^2$, where $\underline{I}$ is some typical value of the intensity of the light to be tested. A suitable value of $\underline{N}$ is then selected. Calibration to make the photometer direct reading is completed by one of the three following procedures, depending on the photometer circuit used.

#### a. External Shunt Circuit

A diagram of this circuit is shown in figure 9. In this circuit

$$\delta_s = \frac{R_L S k I_s}{D_s^2} \qquad (6)$$

where $S$ is the sensitivity of the photometer circuit and $R_L$ is the resistance of the shunt.

Calibration, therefore, requires that, with the photocell illuminated by light from the standard lamp, the external shunt resistance is set so that the potentiometer indicates the value $\delta_s$ given in equation (5). The other parameters of the calibration are usually chosen so that the shunt resistance will be of the order of a few ohms. This order of resistance is used as it is large enough to be set accurately, and small enough so that the voltage developed across the photocell will not cause the photocell to respond nonlinearly. The practice is to maintain the sensitivity of the recorder at a fixed value of 5 millivolts for full-scale deflection. The sensitivity of the preamplifier is therefore set so that this recorder sensitivity and desired range of resistance may be used.

### b. Phototube with Electrometer Amplifier Circuit

The procedure for calibration is the same as that for procedure a. (See figure 12.) The load resistor on the phototube and the controls of the amplifier are adjusted for the optimum performance range of the amplifier. Also, the output of the amplifier should not exceed 5 milliamperes. Hence, other parameters are adjusted so that $R_L$ is greater than 1 ohm and is less than 5 ohms.

### c. Zero-Resistance Circuit

A diagram of this circuit is shown in figure 13. In this circuit, if the photometer is balanced so that no current flows through the galvanometer, then

$$i = i_a a / r_x \qquad (7)*$$

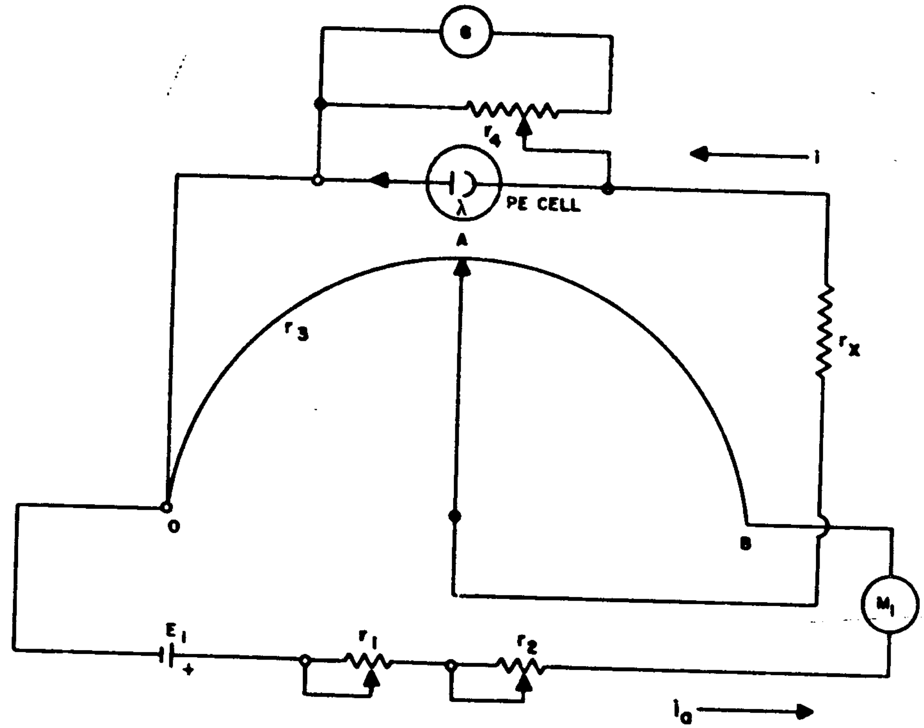
where $i$ is the photocell current,
$i_a$ is the current through the slidewire between 0 and A(figure 13),
$a$ is the resistance of the slidewire between 0 and A (figure 13),
and $r_x$ is the resistance of the resistor, $r_x$.

Assuming the slidewire is graduated from 0 to 100, the reading of the indicator of the slidewire is

$$\delta = a/a_o \qquad (8)$$

---

*This equation is an approximation which depends on $i_a$ being much greater than $i$. In practice, $i_a$ is kept at about 10 milliamperes, and the range of $i$ is from 1 to 20 microamperes. If $i$ is 20 microamperes, the error resulting from the use of this approximation will be 0.3%. For larger values of $i$, a correction in the calibration can be made.[6]




| | |
|---|---|
| $r_1$ | 100-ohm rheostat |
| $r_2$ | 5-ohm rheostat |
| $r_3$ | (OAB) 10-ohm precision potentiometer or slidewire |
| $r_4$ | galvanometer sensitivity control, resistance equal to critical damping resistance of galvanometer |
| $r_x$ | 4-decade resistance box 0-100,000 ohms |
| E | 1 1/2-volt dry battery |
| $M_1$ | milliammeter, 10ma full scale, 30 ohms resistance. |
| G | galvanometer, sensitivity about 0.004 μamp/mm |

Figure 13 "ZERO-RESISTANCE" PHOTOMETER CIRCUIT where $a_o$ is the total resistance of the slidewire (the resistance between 0 and B in figure 13).

Then, combining (1), (7), and (8),

$$\delta = \frac{r_x kI}{i_a a_o D^2} \tag{9}$$

In the calibration of the zero-resistance circuit, $i_a$ is usually kept constant and $r_x$ is varied.

When the photocell is illuminated by light from the standard lamp, $r_x$ is adjusted to obtain a zero reading of the galvanometer when the slidewire is set at the value $\delta_s$ of equation (5) for a given test distance, D. With the photometer thus calibrated, the intensity of the test light is given by equation (2).

### d. Special Procedures

While photometric data are usually presented for a test light operating under the design condition, photometry of the test light under operating conditions other than the design condition is often desirable. Equation (5) can be generalized, taking into account this condition as well as the transmittance of any filters or sector disks used in calibrating, so that

$$\delta_s = \gamma_c \gamma_s F I_s \frac{D^2}{N D_s^2} \tag{5a}$$

where $\gamma_c$ is the transmittance of the color filter at the color temperature of the standard lamp, $\gamma_s$ is the transmittance of the sector disk, and F is the ratio of the output of the light under test when it is operated under the design conditions to the output of the light when it is operated under test conditions. This ratio may be, for example, the ratio of the rated lumen output of the test lamp to the output of the lamp at the test voltage. It also may be the ratio of the intensity in a given direction at the operating voltage to the intensity in this direction at the test voltage.

In the case of lights which are flashed in service but on which photometric measurements are made with the light burning steadily at a selected voltage, the factor F is the ratio of the effective intensity of the flash in a given direction to the steady intensity at the selected voltage in this direction of view.

## 4. TESTING PROCEDURE

The photometer is calibrated as described under "Calibration Procedure" (Section 3.4) to an illuminance range determined by the intensity of the light being tested, the test distance, and the information desired.

The test unit is mounted on the goniometer and is aligned. The angular settings of the goniometer are adjusted so that the origin of the goniometer settings will correspond to the desired axis. This axis usually is chosen with respect to either the seating plane of the unit or some characteristic of the beam such as its peak.

The baffling for stray light is put into place. The eye is placed in the position normally occupied by the photosensor. Examination can then be made to insure that the baffling is properly placed so that no obstructions exist between the light and the photosensor and so that reflections from the walls, floor, and ceiling of the range are intercepted before they reach the photosensor. When the 279-meter range is used, the baffling tube is periodically checked to insure that it is not being obstructed by birds.

If a sealed-reflector lamp is being photometrically measured, the lamp is usually operated at either rated voltage or rated current. Other lamps, such as those used in combination with an optical system, are usually operated at or corrected to rated lumen output. Power for the test and standard lamps is usually obtained from storage batteries, which are periodically recharged. Voltage and current are measured on a potentiometer, and photometric measurements are not made until the lamp has reached stability.

If the goniometer is to be motor driven, the gear ratios are chosen so that the traverse will be slow enough to insure the accurate recording of the characteristics of the light.

## 5. ACKNOWLEDGMENT

The contributions of the many members of the Photometry and Colorimetry Section of the Metrology Division, NBS, who through the years have developed procedures and instrumentation described here is gratefully acknowledged.

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# Review of Elementary Theory
# Of the Photometry of Projection Apparatus

By C. A. Douglas

*Equations based upon simple geometric relations are developed for the illuminance* produced by a projector such as a searchlight, beacon, or floodlight at a distance from the projector. When the beam is rotationally symmetrical but not collimated and the image, virtual or real, subtends a smaller angle at the point of observation than does the objective of the projector, illuminance varies inversely as the square of the distance to the image. If the angle subtended by the image is larger than that subtended by the objective, the illuminance varies inversely as the square of the distance to the objective. The distance at which the two angles are equal is defined as the critical distance. Equations relating critical distance to the radius of the source, the radius of the objective, and the magnification of the system are developed. Approximations for use when the beam of the projector is asymmetric are developed. Very good agreement was found between the computed variation of illuminance with distance and the measured variation of illuminance with distance for a projector forming a virtual image 150 feet behind the objective.*

## 1. Introduction

THE luminous intensity, $I$, of a projector is obtained by measuring the illuminance, $E$, on a surface at a distance, $d$, from the projector and computing $I$ from the relation

$$I = E\,d^2 \qquad (1)$$

The use of this relation without qualification implies that the value of $I$ is independent of the distance at which the illuminance, $E$, is measured.

If measurements could be made at increasing distances, in a perfectly transmitting atmosphere (or in any atmosphere if corrections are made for atmospheric losses), of the illuminance produced by a projector emitting a collimated beam, the value of the product $Ed^2$ increases and approaches a limiting value. The relation

$$I = \lim_{d \to \infty} E\,d^2 \qquad (2)$$

may be considered as the definition of $I$.

In practice it is usually found for these projectors that beyond a certain distance, the critical distance, there is no measurable change in the intensity computed by means of Equation (1).**

The concept of the intensity of a projector producing a collimated beam is valid, and useful, only when this critical distance is exceeded, for then and only then can illuminance at one distance be computed from measurements of illuminance at another distance.

A paper presented at the National Technical Conference of the Illuminating Engineering Society, August 21 to 26, 1966, Minneapolis, Minn. AUTHOR: National Bureau of Standards, Washington, D. C.

*Illumination.

**Such terms as "minimum inverse-square distance" and "limiting distance for the application of the inverse-square law" are frequently used for this distance. The term "critical distance" is used throughout this report as it is a brief, convenient term.

In the photometry of projectors, the photometric distance is made greater than this critical distance whenever it is feasible to do so. Hence, determination of the critical distance is important. The relation between critical distance for a collimated-beam projector and the dimensions of the projector and the source has been extensively treated.[1-3] For a parabolic reflector with a spherical source at the focus, the critical distance along the axis of the reflector is

$$d_c = [RF/r]\ [1 + (R/2F)^2] \qquad (3a)$$

and for a disk source

$$d_c = [RF/r]\ [1 + (R/2F)^2]/[1 - (R/2F)^2] \qquad (3b)$$

where

$d_c$ is the critical distance,
$R$ is the distance from the axis to the edge of the reflector,
$r$ is the radius of the source, and
$F$ is the focal length of the reflector.

Similar equations may be written for a spherical lens.[4]

The relation given in the *IES Lighting Handbook*, Third Edition,[5] may be written as

$$d_c = Rd_e/r \qquad (3c)$$

where $d_c$ and $R$ are as before, and $d_e$ is the distance from the focal point to the edge of the reflector.

For each of these conditions, $d_c$ is given approximately by

$$d_c = RF/r. \qquad (4)$$

The error in computing $d_c$ from this relation is insignificant in most practical applications, particularly when $r$ is small in comparison to $R$, and $F$ is two or more times $R$. Although many projectors are intended for use as collimated-beam projectors, with the source at the focus, there are many types of projectors where this is not the case. For example, in the lens cells of the Fresnel-lens optical landing system, the source is between the focus and the lens so that the lens forms a virtual image of the source 150 feet behind the lens. Spread lenses used in conjunction with collimated beams form either real images in front of the spread lens or virtual images behind the spread lens. The lens cells of the Fresnel-lens optical landing system include a 50-degree horizontal spread lens.

The purpose of this paper is to present an analysis of the factors affecting critical distances and the relation between illuminance and distance of noncollimated beam projectors. Throughout this paper rigor and detailed accuracy of analysis have been subordinated in order to obtain simple, easily interpreted relations free from the complicating and often confusing effects of correction terms. The relations developed are therefore approximations only. They are, however, sufficiently accurate to be used in a qualitative analysis of the performance of a projector system. The performance of the usual types of projector systems is significantly affected by such factors as errors in the shape of the optics and by the variations in illuminance across the surface of the source. Such factors vary from unit to unit and cannot be readily treated in a theoretical analysis. Quantitative analyses of projector systems can, and should, be made by direct measurement on a photometric range.

## 2. Case 1. Projectors Having the Source Between the Focus and the Objective

### 2.1 Illuminance Produced by a Positive Lens

Consider the optical system shown in Fig. 1. The system is symmetric about the line $XX'$ and consists of a positive lens of focal length $F$ and radius $R$. (Throughout this analysis $F$ is assumed to be large in comparison to $R$.)

A source $S$ of radius $r$ and of uniform luminance $L$ is placed at a distance $u$, less than $F$, from the lens.

**SYMBOLS**

| | |
|---|---|
| $L$ | Luminance of source $S$ |
| $d$ | Distance between the projector and the point of observation or measurement |
| $d_c$ | Critical distance |
| $E$ | Illuminance at the point of observation or measurement |
| $F$ | Focal length of the optic of the projector |
| $I$ | Intensity of the projector |
| $I_s$ | Intensity of the source $S$ |
| $I_i$ | Intensity of the image $S'$ of the source |
| $m$ | Magnification of optical system |
| $R$ | Radius of the objective (lens or reflector) when the objective is the field stop of the projector |
| $r$ | Radius of the source $S$ |
| $r'$ | Radius of image $S'$ of source $S$ |
| $S$ | Light source (assumed to be of uniform luminance) |
| $S'$ | Source image formed by the projector |
| $\tau$ | Transmittance of the optical system, if lens type; reflectance of the optic, if reflector type |
| $u$ | Distance between the source and the objective |
| $v$ | Distance between the image and the objective |
| $y$ | Radius of flashed zone on the objective |
| $\theta$ | The beam spread produced by a spread lens |
| $\bar{y}$ | Dimension of asymmetric source in $x$-$y$ plane |
| $\bar{Y}$ | Dimension of asymmetric optic in $x$-$y$ plane |
| $\bar{z}$ | Dimension of asymmetric source in $x$-$z$ plane |
| $\bar{Z}$ | Dimension of asymmetric optic in $x$-$z$ plane |

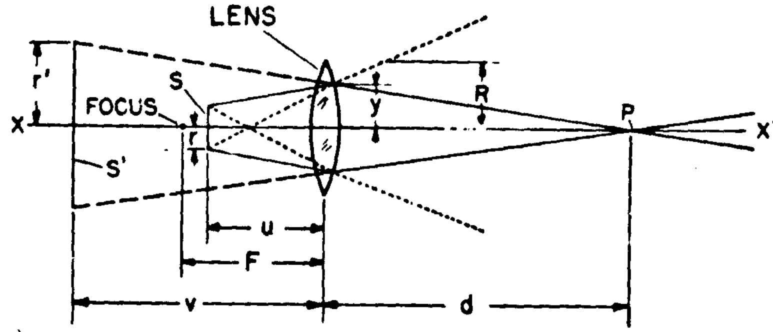


*Figure 1. Optical system with the source between the focus and a positive objective (lens). (Not to scale.)*

The system will form a virtual image $S'$ of the source at a distance $v$ from the lens. Assume the observer's eye, or the photometer, is at the point $P$, a distance $d$ from the lens. With a lossless optical system the luminance of the virtual image, $L'$, would be equal to the luminance of the source, $L$. With a real optical system the luminance

$$L' = L\tau \qquad (5)$$

where $\tau$ is defined as the transmittance of the optical system. The intensity of the source is given by

$$I_s = \pi r^2 L, \qquad (6)$$

and the intensity of the image

$$I_i = \pi (r')^2 L\tau \qquad (7)$$

or

$$I_i = I_s m^2 \tau \qquad (8)$$

When the virtual image, $S'$, is viewed through the lens from the point $P$ and $d$ is less than a critical distance, $d_c$, to be determined later, the entire virtual image will be seen. The illuminance at $P$ will then be

$$E = I_i/(v + d)^2. \quad (d \leq d_c) \qquad (9)$$

Since Equation (9) is of the form

Illuminance = Constant/Distance$^2$,

$I_i$ can be considered to be, by definition, the intensity of the projector applicable to this condition of view with distance to be measured from the position of the virtual image, not from the lens.

From Fig. 1 it is evident that from the point $P$ a zone on the lens of radius $y$ will be flashed, that is, appear to be the source of light, and that the illuminance at $P$ is also given by

$$E = \pi y^2 L\tau/d^2. \quad (d \leq d_c) \qquad (10)$$

If the radius of the virtual image, $r'$, is less than the radius of the lens, the flashed zone will never fill the lens however large $d$ is made. For this condition, Equations (9) and (10) are valid for all distances. However, if $r'$ is greater than $R$ and the distance, $d$, is equal to or greater than a critical distance, $d_c$, the entire lens will be filled and the illuminance at a point on the axis will then be given by

$$E = \pi R^2 L\tau/d^2 \quad (d \geq d_c) \qquad (11)$$

or

$$E = I/d^2 \quad (d \geq d_c) \qquad (12)$$

where

$$I = \pi R^2 L\tau. \qquad (13)$$

$I$ is a constant and hence is by definition the intensity of the projector, applicable to all distances equal to or greater than the critical distance.

Note that when $d$ is equal to $d_c$, Equations (8) and (11) are both valid. Hence at this distance

$$I = I_i d_c^2/(v + d_c)^2.$$

This leads to the somewhat surprising conclusion that at one distance it is sometimes possible for a projector to have two intensities depending upon whether the source is considered to be located at the objective or at the virtual image position.

Note that in practice it is seldom feasible to determine $I_i$ or $I$ to the desired degree of accuracy by Equations (6), (7), (8) or (13). Instead, the illuminance is measured at a known distance from the projector and Equation (9) or Equation (12) is used as applicable.

### *2.2 Determination of the Critical Distance*

Note from Equation (11) that for distances greater than the critical distance the axial illuminance varies inversely as the square of the distance to the lens and is independent of the position of the source with respect to the focus. However, the critical distance, $d_c$, is not independent of the position of the source with respect to the focus. It may be determined as follows.

If there is a critical distance, at this distance

$$y = R. \quad (d = d_c)$$

Therefore, since

$$y/d = r'/(v + d),$$

and

$$r'/r = v/u = m,$$

$$R = rvd_c/u(v + d_c)$$

or

$$R = rmd_c/(v + d_c).$$

Hence

$$d_c = Ru/(r - uR/v), \qquad (14a)$$

and

$$d_c = Rv/(rm - R). \qquad (14b)$$

Note that as $u$ approaches $F$, $v$ becomes infinite, and Equation (14a) becomes the familiar relation used for collimated-beam projectors,

$$\lim_{u \to F} d_c = RF/r. \qquad (4)$$

From Equations (14a) and (14b), note that $d_c$ becomes infinite when

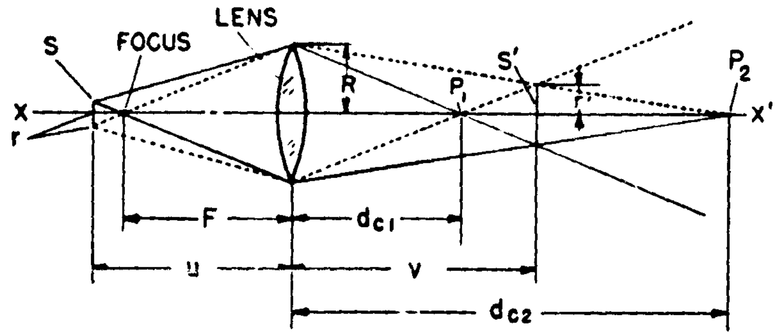


*Figure 2. Optical system with the focus between the source and a positive objective (lens).*

$$r = Ru/v = R/m, \quad (15)$$

for then the radius, $r'$, of the virtual source is just equal to the radius of the lens, $R$, and the lens will appear flashed only when viewed from an infinite distance. If $r$ is less than $R/m$, the value obtained for $d_c$ is negative indicating that $r'$ is less than $R$ and that the virtual source will not completely fill the lens at any viewing distance. Hence, under this condition Equation (9) is valid for all distances.

## 3. Case 2. Projectors Having the Focus Between the Source and the Lens

### *3.1 Illuminance Produced by a Positive Lens*

If the distance from the source to the lens, $u$, is greater than the focal length of the lens, $F$, the system will form a real image of the source a distance $v$ from the lens, as shown in Fig. 2.

Consider first the case in which the radius of the image, $r'$, is less than the radius of the lens. It is evident from Fig. 2, on which the limiting rays from the edge of the lens are shown, that the entire lens will appear to be flashed only between the points $P_1$ and $P_2$. The illuminance at a point on the axis is, as for the flashed lens of Case 1,

$$E = \pi R^2 L_T/d^2. \quad (d_{c1} \leq d \leq d_{c2}) \quad (11)$$

The inverse square law holds with distance measured from the lens to the point of observation, or measurement, and as before

$$I = \pi R^2 L_T. \quad (d_{c1} \leq d \leq d_{c2}) \quad (13)$$

When the lens is observed from $P_4$ (see Fig. 3) where $d$ is equal to or greater than $d_{c2}$, the entire image will be visible and the illuminance at $P_4$ is given by

$$E = I_i/(d - v)^2. \quad (d \geq d_{c2}) \quad (16)$$

If the lens is viewed from a point $P_3$ on the axis between the point $P_1$ and the lens, the entire lens will not be flashed, for rays from the outer part of the lens to the image cross the axis at or beyond the point $P_1$. Instead, a zone of radius $y$ will appear flashed, as is evident from the limiting rays shown in Fig. 4. The illuminance at $P_3$ will be

$$E = \pi y^2 L_T/d^2. \quad (d \leq d_{c1}) \quad (17)$$

Since

$$y = r'd/(v - d), \quad (18)$$

$$E = \pi (r')^2 L_T/(v - d)^2 \quad (d \leq d_{c1}) \quad (19)$$

or

$$E = I_i/(d - v)^2. \quad (d \leq d_{c1}) \quad (20)$$

Note that $(v - d)$ and $(d - v)$ are both the distance between the point of observation and the image formed by the lens.

### *3.2 Determination of the Critical Distances For Lenses Forming Real Images*

If $r'$ is less than $R$, the critical distances, $d_{c1}$ and $d_{c2}$, may be found by noting that for observations from $P_3$ (Fig. 4),

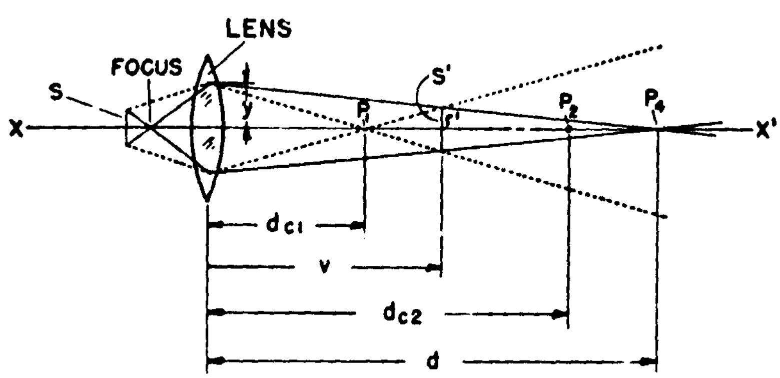


*Figure 3. Flashed zone of an optical system forming a real image with the image between the point of observation and the objective.*

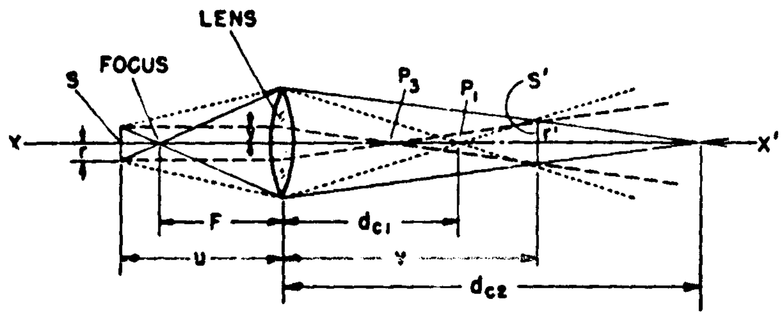


*Figure 4. Flashed zone of an optical system forming a real image with the point of observation between the objective and the image.*

$$y = rmd/(v - d) = rd/u\,(1 - d/v) \quad (21)$$

and substituting $R$ for $y$, and $d_{c1}$ for $d$ obtaining

$$d_{c1} = Rv/(R + mr) \quad (22a)$$

or

$$d_{c1} = Ruv/(Ru + rv). \quad (22b)$$

Similarly, for observations from $P_4$ (Fig. 3)

$$d_{c2} = Rv/(R - mr) \quad (23a)$$

or

$$d_{c2} = Ruv/(Ru - rv). \quad (23b)$$

Since $r' = mr,$

when $r'$ is equal to or greater than $R$, $d_{c2}$ becomes infinite, and the lens is flashed for all distances greater than $d_{c1}$. For this condition, Equation (11) is applicable for all distances greater than $d_{c1}$.

When the source is placed at the focus,

$$u = F,$$

$m$ and $v$ become infinite, and Equation (22) reduces to Equation (4). That is,

$$d_{c1} = RF/r, \quad (4)$$

and $d_{c2}$ becomes infinite.

### 3.3 *Measurement of Beam Angles*

When the optical system forms an image, virtual or real, of the source instead of producing a collimated beam, and the point of observation or measurement is at a distance such that the objective of the system is not completely flashed, angles of observation as well as the illuminance are determined, using the distance between the image and the point of observation as a base, *not* the distance between the objective and the point of observation. That is, the vertex of the angle between the line of sight and the axis of the light is located at the image of the source, not at the objective of the projector. However, when a light is mounted on a goniometer, it is usually more convenient to measure angles whose vertices are at the projector. The relation between the two angles is easily found by the use of Fig. 5.

Since

$$\tan \psi = a/d$$

and

$$\tan \phi = a/(d + v),$$

for small angles

$$\psi \doteq a/d,$$

and

$$\phi \doteq a/(d + v),$$

where $a$ is the distance from the point of observation to the axis $XX'$.

Hence,

$$\phi \doteq [d/(d + v)]\psi, \quad (24a)$$

or

$$\phi \doteq [d/(d + mu)]\psi. \quad (24b)$$

### 3.4 *Asymmetric Beams*

Throughout the preceding discussion it has been assumed that the optical system is rotationally symmetric about the optical axis of the system. Thus, a disk or spherical source and an optical system com-

*Figure 5. Correction of angles of an optical system forming a virtual image.*

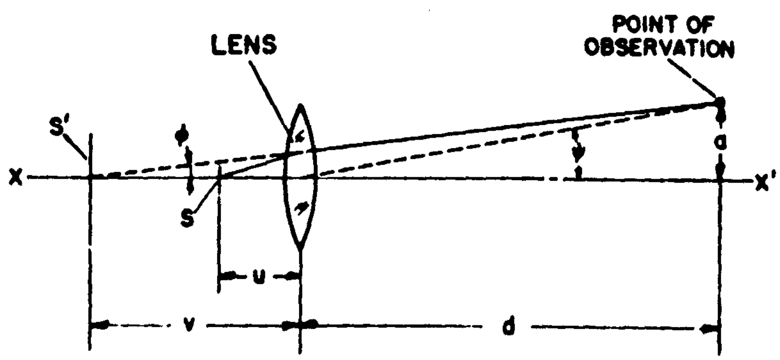

**Table 1—Photometric Measurements of an Asymmetric Projector**

| PHOTOMETRIC DISTANCE ($d$) (FEET) | PEAK INTENSITY ($I$) (KILOCANDELAS) |
|---|---|
| 30 | 32.0 |
| 100 | 31.3 |
| 920 | 34.8 |

posed of spherical elements have been assumed. In such systems the beam pattern is the same in every plane containing the axis of the optical system.

However, lights having unequal vertical and horizontal beam spreads are often required. These unequal beam spreads may be obtained by using sources in which the width (the dimension in the $z$ direction) and the height (the dimension in the $y$ direction) are not equal and/or by adding cylindrical elements, with the axis of the cylinders parallel to the $y$ or $z$ axis. When cylindrical elements are used, the image of the source formed by a narrow section in the $y$ direction through the center of the lens will not be the same distance from the lens as the image formed by a narrow section in the $z$ direction. That is, the projection of the image of the source on the $x$-$y$ plane will not be in the same position as the projection of the image on the $x$-$z$ plane. Under these conditions the equations of the type given below will give approximate values. The applicable equations of Section 2.2 are applied to determine if the viewing, or test, distance exceeds the critical distance using the dimensions of the projection on the $x$-$y$ and the $x$-$z$ planes, in turn, to determine $d_{ey}$ and $d_{ez}$, respectively.

Thus, from Equations (14a) and (14b), if the projection system forms a virtual image,

$$d_{ey} = \bar{Y} u_y / (\bar{y} - u_y \bar{Y} / v_y), \qquad (25a)$$

or

$$d_{ey} = \bar{Y} v_y \, (\bar{y} m_y - \bar{Y}) \qquad (25b)$$

where

$\bar{Y}$ is the dimension of the lens in the $x$-$y$ plane,
$\bar{y}$ is the dimension of the source in this plane,
and the subscript $y$ refers to quantities measured in the $x$-$y$ plane.

Similar equations can be written for $d_{ez}$ where $\bar{Z}$, $\bar{z}$, and the subscript $z$ replace $\bar{Y}$, $\bar{y}$, and the subscript $y$.

If the observation distance is less than the critical distance in both the $x$-$y$ and the $x$-$z$ planes, that is,

$$d \leq d_{ey}$$

and

$$d \leq d_{ez},$$

Equation (8) may be written as

$$I_t = I_s m_y m_z \tau, \qquad (26)$$

and Equation (9) may be written as

$$I_t = E \, (v_y + d) \, (v_z + d). \qquad (27)$$

Equation (27) may be considered as the definition of intensity under these conditions.

Similarly, if

$$d \leq d_{ey}$$

and

$$d \geq d_{ez},$$

then

$$I_t = E \, (v_y + d) \, (d). \qquad (28)$$

Similar equations can, of course, be written interchanging the $y$'s and the $z$'s.

Similar expressions can, of course, be developed for systems which form real images in either or both planes by using the terms $(v - d)$ or $(d - v)$ in place of $(v + d)$, as applicable.

### 3.5 Experimental Confirmation

The principles developed in Section 3.4 were applied to a cell of the Fresnel-lens optical landing system. The cell has a lens with vertical cylindrical elements forming a real image of the source ahead of the unit to produce a wide horizontal spread, with the following dimensions (approximate):

$\bar{Z}$ = 0.16 inch ($2\bar{Z}$ is the width of one cylindrical element)
$\bar{z}$ = 0.38 inch
$v_z$ = 0.5 inch (computed from design beam spread of 40 degrees)
$u_z$ = 24 inches.

From these dimensions it is apparent that the first critical distance in the $x$-$z$ (horizontal) plane, $d_{ez1}$, is very small. (This is usually the case in the plane in which there is an appreciable spread.)

From Equation (22b),

$$d_{ez1} = 0.48 \text{ inch}$$

The second critical distance in the horizontal plane is also very small. From Equation (23b),

$$d_{ez2} = 0.52 \text{ inch}$$

In the $x$-$y$ (vertical) plane, the optical system forms a virtual image of the source 150 feet behind the lens. Pertinent dimensions are as follows:

$\bar{Y}$ = 0.4 foot approximately
$\bar{y}$ = 0.003 foot approximately
$v_y$ = 150 feet
$u_y$ = 2 feet
$m_y$ = 75
$r'_y = m_y \bar{y}$ = 0.2 foot

Since $r'_y$ is less than $\bar{Y}$, the lens will never be completely flashed in the $y$ direction.

The relation between illuminance produced by the system and the luminous "intensity" is

$$I = E \, (v_y + d) \, (d - v_z) \qquad (29)$$

or

$$I = E \, (150 + d) \, (d - 0.04)$$

where $d$ is in feet.

Or, since $v_z$ is very small compared to typical values of $d$,

$$I = E \, (150 + d) \, (d) \qquad (30)$$

Measurements were made at three distances of the illuminance produced by a Fresnel-lens optical landing system cell and the luminous intensity was computed by Equation (27). Results are given in Table I. The consistency of the values of $I$ is surprisingly good.

## 3.6 Practical Considerations

### 3.6.1 Simplified Equations

Although the equations which have been developed describe the performance of a projector, they are in forms which often are too complex to be used to best advantage by the photometrist, who, given the task of determining the intensity distribution of a projector, desires only sufficient information to permit him to choose a photometric distance greater than the critical distance. Often he is given very little information concerning the design parameters of the projector. The relations given below contain only parameters which are easily measured.

Consider first a projector which forms a virtual image of the source. From Equation (14a) it is evident that

$$d_c \geqq Ru/r \qquad (31)$$

Hence, if the photometric or observation distance is less than $Ru/r$, that is, if

$$d < Ru/r \qquad (32)$$

the entire lens is not flashed and illuminance varies inversely as the square of the distance to the image.

The objective of a projector forming a real image of the source will appear flashed at all distances equal to or greater than the image distance if (and only if) the radius of the image is equal to or greater than the radius of the objective. Hence,

$$r' = rv/u \geqq R$$

or

$$v \geqq Ru/r. \qquad (33)$$

(The distance $v$ can usually be determined readily from direct observation of the position of the image.)

Hence, when a real image is formed at a distance equal to or greater than $Ru/r$, illuminance varies inversely as the square of the distance from the objective for all distances greater than the image distance, $v$. It is apparent from simple geometric relations (see Fig. 2) that, if the condition of relation (33) is met, the objective will be flashed for distances in the range $v/2$ to $v$ inclusive also.

### 3.6.2 Application to Spread Lenses

Projectors often contain lens elements to increase the beam spread of the light from the projector. These lens elements form images, real or virtual, which are too small to flash the element. Hence, distance should be measured from the position of the image. (Note these elements usually are asymmetric and the image distances in the $y$ and $z$ planes are different. For simplicity, only one plane is treated here.) The geometric relations of these spread elements are illustrated in Fig. 6.

The angle $\theta$, in degrees, is commonly called the beam spread of the lens. The relation between image distance and beam spread is

$$v = (b/2) \text{ ctn } \theta/2, \qquad (34)$$

where $b$ is the width of the spreader elements. The following approximation is sufficiently accurate for most purposes.

$$v \sim 60b/\theta \qquad (35)$$

when $\theta$ is in degrees.

Note that when a spread lens is used, each spreader element acts as a separate projector, and that usually these elements will not be completely flashed at practical observation distances. Under these conditions, distance should properly be measured from the images produced by these spreader elements. With the usual projectors, $v$ is so small in comparison to $d$ that no significant error results when the term $(v + d)$ is replaced by $d$. For example, consider a five-degree spread lens with cylindrical elements one inch wide. From Equation (35), $v$ is found to be 12 inches. As this is a rather extreme example, $v$ will usually be considerably less than this. The criteria determining the photometric distance are then:

1. The photometric distance, $d$, should be large in comparison to the image distance $v$.

2. The angular subtense of the light unit from the point of measurement should be small in comparison to the beam spread of the light.

### 3.6.3 Incandescent Lamps as Sources

The filament of an incandescent lamp is not a source of uniform luminance. Hence, the overall dimensions of the filament should not be considered as the size of source in determining the critical distance.

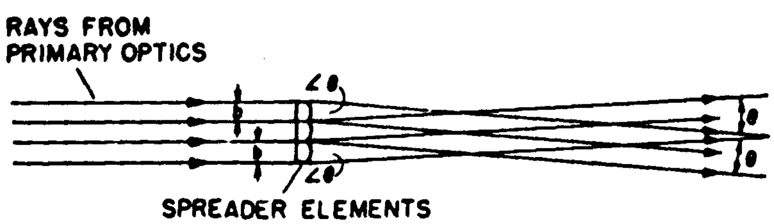


*Figure 6. Geometric relations of a spreader element.*

Instead, the size of the smallest separable element of the filament should be used.[5] The smallest separable element will usually be the filament wire itself but at times it will be the helix. The dimensions of both the filament wire and the helix must be considered, as the separate turns of the filament wire may be discernible from the central part of a reflector while only the helix may be discernible from the outer part of the reflector.[6]

#### *3.6.4 Application to Objectives Of Small f-Number*

The development of the equations of this report is based on the assumption that the radius of the objective is small compared to its focal length. In practice, the radius of the objective is often as large as or larger than the focal length. Frequently, when applied to projection systems of the latter type, these equations yield results sufficiently accurate for engineering purposes if $u$ and $v$ are measured to the most distant part of the objective instead of along the axis of the projection system.[2, 6]

## Conclusion

A review of the elementary theory of the photometry of projectors has been made. Use was made of simple geometric relations to develop equations relating distance with the illuminance produced by projectors. Particular attention has been given to projectors producing noncollimated beams and to the effect of spreader elements in the optical system. Equations have been developed relating critical distance to the radius of the source, the radius of the objective and the magnification of the system. Equations applicable to asymmetric beams have been stated. The agreement between values computed from these relations and direct measurements is very good.


## Acknowledgment

The photometry of the Fresnel-lens optical landing system was performed by my colleague A. C. Wall.


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# Photometer for Measurement of Effective Intensity of Condenser-Discharge Lights

By CHARLES A. DOUGLAS

## 1. Introduction

In the past, measurements of the effective intensity of condenser-discharge lights have generally been made by two methods: (1) coupling the output of a phototube to a cathode-ray oscilloscope, photographing the trace of the instantaneous intensity against time, integrating the area under the curve and computing the effective intensity; and (2) charging a capacitor by the photoelectric current generated by one or more flashes and measuring the voltage developed with a vacuum-tube voltmeter. Both of these methods are time consuming when intensity distribution measurements of a projector with a flashing source are being made. In addition, when the source is a condenser-discharge lamp, there has often been uncertainty about the accuracy of the correction of the spectral response of the photometric system to the CIE standard observer luminosity function; the sensitivity may be so low that short photometric distances are required; although the average photoelectric current is low, peak currents are high, and, unless suitable precautions are taken, the phototube may be saturated during part of the flash, destroying the linearity of the system. A method that will allow measurements of the effective intensity distribution of condenser-discharge units producing repetitive flashes to be made and recorded automatically is very desirable. The development of such a method, designed to avoid difficulties of the types described above and making use of commercially available equipment, wherever possible, is described below.

## 2. Effective Intensity Photometer

The effective-intensity photometric system is shown schematically in Fig. 1. The test and standard distances, the intensity of the standard lamp, and the transmittance of the sector disc are chosen so that the average illumination on the photocell during calibration is of the same order as the illumination from the projectors under test. The sector disc was used to obtain flashes during calibration which had roughly the same intensity-time relation as the flashes from the lights under test. (As will be shown below, this latter condition is not necessary.) With this system it is possible to obtain in approximately three minutes an intensity distribution curve of a projector using a condenser-discharge lamp which produces repetitive flashes.

Light from either the test unit or the standard lamp falls on a diffusing glass so that the distribution of illumination on the photosensitive surface of the phototube is independent of the distance of the light source. A small aperture in front of this glass is used to control the illumination on the phototube. The light then passes through the luminosity filter to the phototube. This filter is so designed* that the spectral response of the phototube-filter-diffusing glass combination is essentially that of the luminosity function of the CIE standard observer.[1]

A type PJ-14B phototube is used because it is stable, it has a low dark current, and its relatively flat spectral response simplifies the design of the luminosity filter. The correction for the spectral sensitivity of the phototube by the luminosity filter is sufficiently accurate for the photometry of any non-chromatic ("white") light using an incandescent lamp or a conventional flashtube. The luminous sensitivity of the phototube-filter combination is sufficiently high so that the photometric distance may be made as large as necessary.

The output current of the phototube is smoothed by the resistance-capacitance network so that the phototube will not be saturated and the d-c electrometer-amplifier will not be overloaded during the flashes and so that output current will be sufficiently stable to produce a smooth curve on the recording potentiometer. The output of the system is, of course, proportional to the average photoelectric current and hence to the average illumination at the phototube.

The General Radio Type 1230A d-c amplifier and electrometer is operated with its "Ground" switch

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*The luminosity filter was designed by Mr. I. Nimeroff of the National Bureau of Standards.

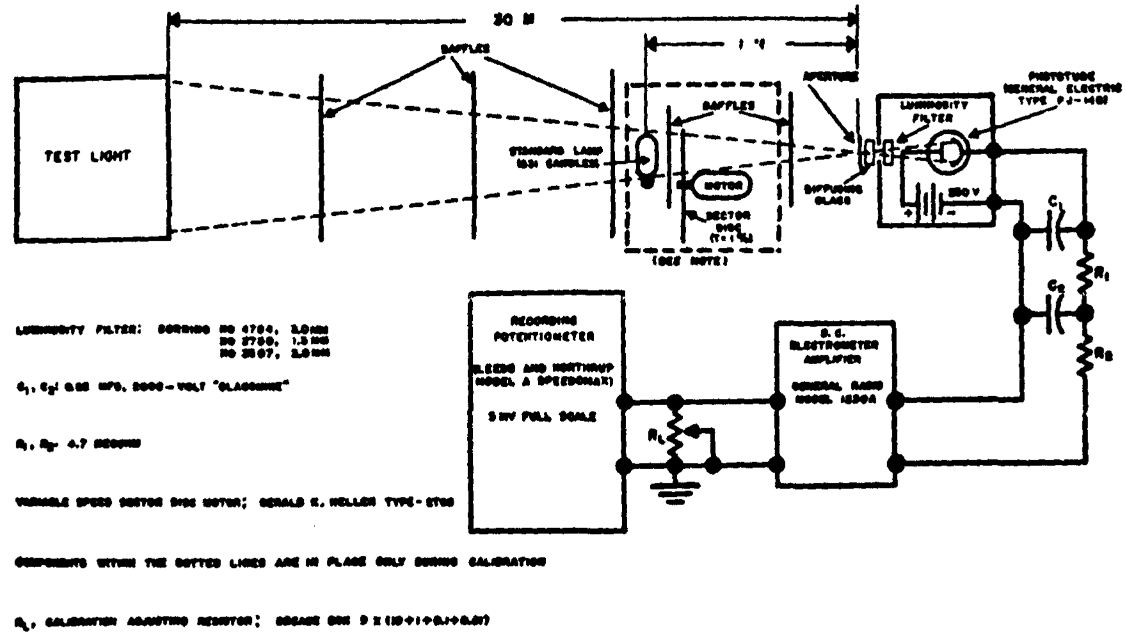


**Figure 1.** **Block diagram of effective-intensity photometric system. Components with dotted lines are in place only during calibration.** $R_L$**, calibration-adjusting resistor, is a decade box adjustable by 0.01-ohm steps to 99.99 ohms.**

in the "I" position. With this arrangement, use of a single capacitor across the input of the instrument is unsatisfactory because the feedback in the instrument produces low-frequency oscillations (of the order of 0.1 to 0.5 cps) in the output. A network of the type shown eliminates these oscillations. With the "Ground" switch in the "E" position, a single capacitor across the input may be used. The time constant of the circuit is then a function of the capacitance across the input and the input resistance selected by means of the "Input Resistance" switch of the electrometer-amplifier. The arrangement shown is considered preferable unless very long time constants are required, for the time constant is relatively independent of the input resistance selected and may be readily adjusted by changing the values of $R_1$ and $R_2$. Ordinary carbon resistors are satisfactory, but capacitors with very high leakage-resistance are required to obtain stability of calibration.

The electrometer-amplifier is designed to operate an external five-milliampere full-scale meter or recorder. However, the recording potentiometer available had a five-millivolt full-scale range. Load resistor $R_L$ is used to obtain a suitable input signal for the recording potentiometer from the output current of the electrometer-amplifier. It also provides a convenient means of making small adjustments of the over-all sensitivity of the photometric system. The minimum value used for $R_L$ is one ohm in order that a full-scale reading of the potentiometer may be obtained before the electrometer is overloaded.

## 3. Theory of Operation

The method of obtaining the effective intensity of a condenser-discharge light flashed at a known rate from measurements of the average intensity is developed below.

### 3.1 *Definitions of Terms and Symbols.*

The terms and symbols used in this paper are defined as follows:

- $I$ the instantaneous intensity of the test light during a cycle
- $\bar{I}$ the average intensity of the test light during a complete flash cycle
- $I_e$ the effective intensity of the test light
- $I_s$ the intensity of the standard lamp
- $T$ the transmittance of the sector disc
- $D$ the distance between the test light and the diffusing glass of the photometer
- $d$ the distance between the standard lamp and the diffusing glass of the photometer
- $\tau$ the time in seconds required for a complete flash cycle

*R* the recorder reading for the test source

$R_s$ the recorder reading during calibration with the standard lamp and rotating sector disc.

**3.2** *Basic Relations.*

The effective intensity is defined as

$$I_e = \frac{\int_{t_1}^{t_2} I\,dt}{0.2 + t_2 - t_1}, \qquad (1)$$

where $t_1$ is the time in seconds of the beginning of the flash and $t_2$ is the time of the end of the flash. An equation of this form was first suggested by Blondel and Rey but has rarely been used.[2]

The times $t_1$ and $t_2$ are chosen so that $I_e$ is equal to $I$ at these times.[3] However, if the effective intensity is sufficiently small in comparison to the instantaneous intensity over most of the flash then $\int_{t_1}^{t_2} I\,dt$ may be replaced by $\int_0^{\tau} I\,dt$ without introducing significant errors. In addition, the flash of a condenser-discharge light is so short (generally less than 0.001 second) that $t_2 - t_1$ is negligibly small in comparison to 0.2.

Therefore equation (1) may be rewritten as

$$I_e = 5 \int_0^{\tau} I\,dt. \qquad (2)$$

But

$$\bar{I} = \frac{\int_0^{\tau} I\,dt}{\tau}. \qquad (3)$$

Therefore

$$I_e = 5\,\bar{I}\,\tau. \qquad (4)$$

Note that $\tau$, the period of the flash cycle, must be known, and must be stable, to the same degree of accuracy as is desired for the effective intensity.

The time constant of the input circuit is sufficiently long so that the recorder indication is nearly steady and proportional to the average current of the phototube. During a flash the photoelectric current charges capacitor $C$, and the voltage across the

[redacted]

Combining equations (4) and (5),

$$\frac{I_e}{R} = \frac{5\tau D^2}{k}. \qquad (6)$$

For ease in interpretation of the recorder charts, the ratio $I_e/R$ is made the product of either an integer, or the reciprocal of an integer, and an appropriate power of ten by adjusting the parameters of the circuit to obtain the proper value of $k$. The recorder chart can then be graduated in a convenient number of effective candles per chart division.

**3.3** *Calibration.*

The calibration of the photometric system to obtain the proper value of $k$ is accomplished by means of a standard lamp.

Since

$$R_s = \frac{k I_s T}{d^2}, \qquad (7)$$

then

$$R_s = 5\tau D^2 \frac{R}{I_e} \cdot \frac{I_s T}{d^2}. \qquad (8)$$

A value of $I_e/R$ is chosen and the parameters of the photometric system are adjusted to obtain the desired $R_s$. The aperture of the photometer is kept sufficiently small so that the phototube will not be saturated during a flash. The electrometer-amplifier is generally operated on the 100- or 300-millivolt range using an input resistance of $10^7$ or $10^8$ ohms. The maximum output of the instrument is kept near, but below, the design maximum. Final adjustment of the calibration is obtained by adjusting the value of the Resistor, $R_L$, across the amplifier output until the voltage drop across this resistor drives the recorder to the desired reading, $R_s$.

**3.4** *Performance Tests.*

The photometric system was calibrated by means of a standard lamp one meter from the photometer and two 2-aperture sector discs having transmittances of 0.0098 and 0.0252. These sector discs were

[redacted]

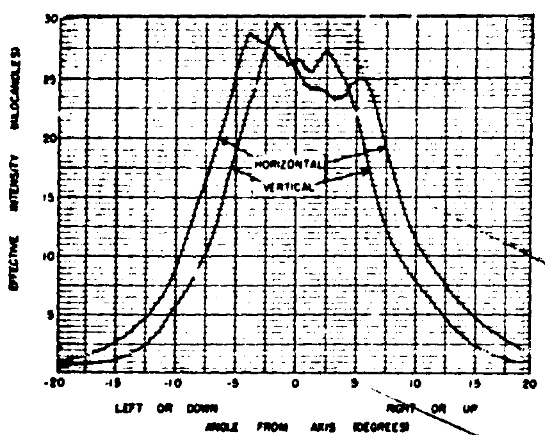


**Figure 2. Typical horizontal and vertical intensity distributions made automatically with the effective-intensity photometer of a projector with a flash rate of two per second.**

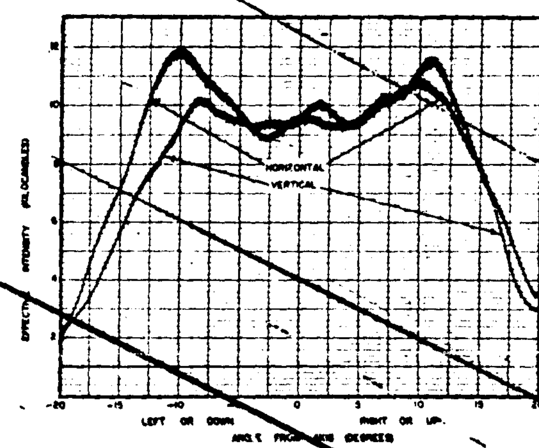


**Figure 3. Typical horizontal and vertical intensity distributions made automatically with the effective-intensity photometer of a projector with a flash rate of one per second.**

the intensity distributions of a number of projectors which use condenser-discharge lamps. The light under test was mounted on a goniometer to permit rotation about a fixed horizontal axis perpendicular to the photometric axis and about a secondary axis perpendicular to the first and initially vertical. These angles are referred to as "vertical" and "horizontal" respectively. Traverses were taken by driving the goniometer and the recorder-chart drives with synchronous motors, thus recording the intensity of the light as a function of the angle from the axis of the light. The speed of the goniometer drive was 7.5 degrees per minute. This speed was sufficiently slow so that the photometric system could follow the changes in intensity even with the long time constant introduced by the smoothing network in the input circuit to the d-c amplifier.

Examples of these measurements are shown in Figs. 2 and 3. As shown in Fig. 2, when the flash rate is two per second, the time constant of the photometric system is sufficient to smooth the curves so that the effect of the individual flashes is not shown. However, as shown by Fig. 3, when the flash rate is one per second, the effect of the individual flashes is visible. This effect could have easily been reduced by lengthening the time constant of the system by increasing the value of $R_1$ and $R_2$. However, the time constant would then have been so long that the scan rate of the goniometer would have had to be considerably slower than the 7.5 degrees per minute rate used when these figures were recorded.

## 4. Discussion

The photometer system described here is, of course, not limited to the photometry of condenser-discharge lights only, but is applicable to any light having a flash duration of less than about one millisecond (between the times when the intensity is about 5 per cent of peak intensity). The effective intensity distributions of lights with flash durations somewhat longer than one millisecond (up to about 0.01 second) can often be recorded automatically with this system if a suitable correction factor is included in the calibration. This factor is the ratio of the effective intensity computed by equation (1) to that computed by equation (2).

The components used in the system described here can, of course, be replaced with components having similar characteristics. Those listed in Fig. 1 were used because they were readily available.

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# Effective Intensity of Flashing Lights

By THEODORE H. PROJECTOR

Flashing lights are in widespread use as signals, markers and warnings because of their distinctiveness and their superiority over steady burning lights in attracting attention. Complete evaluation of flashing lights or lighting systems employing flashing lights usually requires the weighing of many factors such as configuration, background, information to be presented, etc., and may involve complex, elusive, psychological elements. In most applications of flashing lights to land, sea, air navigation or traffic control, one of the fundamental considerations is the apparent or "effective" intensity of the lights. This has been the subject of considerable research for over a century, especially since 1911, when the classical work of Blondel and Rey was published. It is the purpose of this paper to review existing psychophysical knowledge in the field essential to the design and evaluation of flashing lights.

The evaluation of the effective intensity of a flashing light is relatively easy under some conditions and very difficult under others. If the flashing light is viewed under conditions such that the illuminance at the observer's eye is at or very near his visual threshold, then the problem is relatively simple. For threshold viewing, Blondel and Rey defined the effective intensity of a flashing light as the intensity of a steady light which is seen at threshold under the same conditions. The experimental techniques for ascertaining the threshold for a steady light may be used to determine the equivalent threshold for a flashing light. If, however, the flashing light is viewed under conditions in which the illuminance at the eye is above threshold, and especially if it is far above threshold, it is difficult if not impossible to exclude from any judgment as to effective intensity psychological elements which are not present at low levels. Observers find it difficult to distinguish between the judgments "equally intense" and "equally irritating." In spite of this difficulty, the concept of effective intensity has been extended to above-threshold viewing, and has been found useful at least at relatively low levels, that is, up to about twenty times threshold. The experimental technique for above-threshold measurements of effective intensity involves the comparison of a flashing and a steady-burning light, one of which is varied in intensity until the observer reports apparently equal intensity. Such judgments are difficult to make, requiring as they do the equating of profoundly dissimilar appearances. The results of above-threshold measurements indicate that the effective intensity of a flashing light varies with the level of illuminance at the eye, and is thus fundamentally different from the intensity of a steady-burning source, which is a fixed property of a light source, essentially independent of the viewing condition. The concept of "effective intensity" may be useful only at the lower levels of illuminance at the eye where the fundamental ability to see is of great importance, so that the difficulty of making intensity judgments at higher levels may not have practical implications. In any event any measure of effective intensity of flashing lights must be related to the viewing conditions, and the use of the concept should be avoided at high levels of illuminance.

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## Classical Work of Blondel and Rey

The experimental work reported by Blondel and Rey in 1911[1] consisted of a series of determinations of threshold illuminance with the observers viewing a centrally fixated point source of incandescent-lamp light which was flashed with a shutter at controlled durations from 0.001 to 3 seconds. The wave form of the flash was approximately square; that is, the light appeared at full intensity at the onset of the flash, remained at full intensity for the duration of the flash and then fell abruptly to zero at the end of the flash. For each flash duration, the intensity of the light was varied to establish the threshold. The results of the experiments are shown in Fig. 1.

The ordinate of a point is the ratio of the energy of a threshold test flash to the energy of a three-second threshold flash, expressed as $Et/3E_3$, where $E$ is the instantaneous illuminance at the eye during the test flash and $E_3$ is the instantaneous illuminance at the eye during the three-second flash. A three-second flash was chosen for comparison

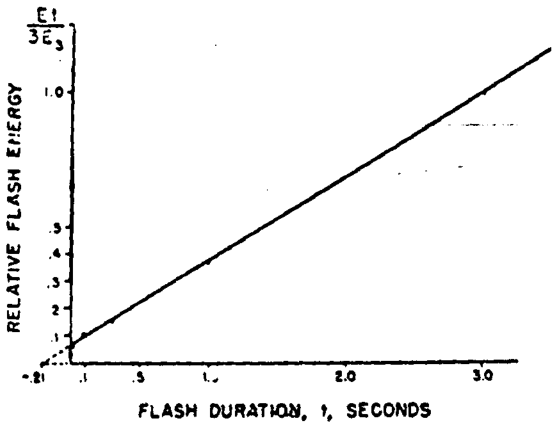


**Figure 1. Ratio of test flash energy required for threshold excitation to energy of a threshold flash of 3 seconds duration as a function of test flash duration. (Blondel and Rey).**

with the test flash as representing a duration sufficiently long that the effective intensity would be closely equal to the intensity of the same lamp burning steadily. The abscissas are the durations of the flashes. Blondel and Rey found that a straight line could be fitted very well to the experimental points plotted on the graph, and that the equation of this line could be expressed in the form:

$$\frac{E_o}{E} = \frac{t}{.21 + t}$$

where $E_o$ is the threshold illuminance for a steady source and $E$ is the illuminance during the time $t$ of a flashing light at threshold. If it is kept in mind that the equation applies to threshold illuminance under a given set of viewing conditions, then intensity may be used interchangeably with illuminance:

$$\frac{E_o}{E} = \frac{I_o}{I} = \frac{t}{.21 + t}$$

and one may refer to the effective intensity as a property of the flashing light.

The Blondel-Rey equation has been investigated extensively since 1911 by Blondel and Rey themselves,[2] and by many others.[3,4,5] (The references cited above and in what follows are not intended to be complete but are selected as illustrative, or as of special importance. Work done prior to 1911 was summarized by Blondel and Rey,[1] and many additional references will be found in the other references given in this paper.) The validity of the equation has been generally well established, not only under the special conditions studied by Blondel and Rey, but also with sources of different sizes,[6] with observers in different states of dark adaptation,[7] with observations made in peripheral areas of the retina,[8,9,10] and with different colors.[8] Research work on the relation between effective intensity and the wave form of the flash, extremely short durations, the level of illuminance at the observer's eye, and flash repetition rate will be discussed in detail later. In addition to the investigations which have been more or less directly connected with effective intensity, there have been a number of studies of more complex visual problems connected with flashing light signals in which effective intensity played an important part.[11,12] Such studies have yielded results consonant with the Blondel-Rey equation.

## Analysis of the Blondel-Rey Equation

The Blondel-Rey equation has been found to have such widespread applicability that it might be well to analyze its implications before discussing its specialized aspects. Its simplest general expression (for flashes of square wave form) is

$$\frac{I_e}{I} = \frac{t}{a + t} \qquad (1)$$

where $I_e$ is the effective intensity, $I$ is the instantaneous intensity during the flash, $t$ is the duration of the flash in seconds, and $a$ is a constant. The value of $a$ found by Blondel and Rey, 0.21, has been confirmed by subsequent work, although the inherent imprecision of experimental work in this field makes it desirable to round the constant to 0.2. As will be shown later, the constant $a$ appears to decrease as the level of illuminance at the eye increases, with the value of 0.2 representing its maximum value at threshold illuminance.

For purposes of analysis, methods of flashing lights may be divided into two categories. In the first, the light is burned steadily, and flashing is obtained by occulting the light with a shutter or similar device so as to conceal the light from the observer except during the flash interval. In this method, the light from the source is not visible during the dark intervals and the luminous energy emitted during the dark interval, as well as the corresponding electrical or other form of energy which produces it, may be considered "wasted." In the second category, the light source is energized only during the "on" interval, as when flashes are produced with an electrical switch, or by discharging the electrical energy stored in a condenser into an electronic flash lamp. Flashes produced by these methods do not involve "wasted" energy although the efficiency of utilization of energy may vary with the type of light source and the method of producing the flash. Flashes produced by oscillating or rotating steady burning light are considered to belong in this second category, since the light energy is fully utilized when, as is usually the case, all the directions swept by the beam are of importance

Fig. 2 gives curves of the Blondel-Rey equation for flashing lights of the first category, where a steady burning light of intensity $I$ is exposed for a time $t$, yielding an effective intensity $I_e$. The effective intensity for extremely short flash durations approaches zero, increases rapidly as duration increases, and finally approaches the steady intensity $I$ asymptotically as durations become relatively long. According to the mathematics of the equation, it would take an infinitely long duration to achieve full equality of $I_e$ and $I$, but for practical purposes the two are equal when the duration is of the order of two or more seconds, depending somewhat on illuminance level. The relation between illuminance level, as represented by a corresponding value of $a$, and effective intensity, is shown in the figure. For example, for a duration of 0.2 seconds, $I_e$ is about 50 per cent of $I$ at threshold when $a = 0.2$, but about 90 per cent of $I$ when the illuminance is such that $a = 0.02$. Thus, if a given flash of light appears equal in intensity to a given steady burning light when both appear to be at threshold, then the flashing light will appear more intense than the steady light when the conditions of observation are such that they both appear to be well above threshold. This effect may explain a number of apparently anomalous field observations

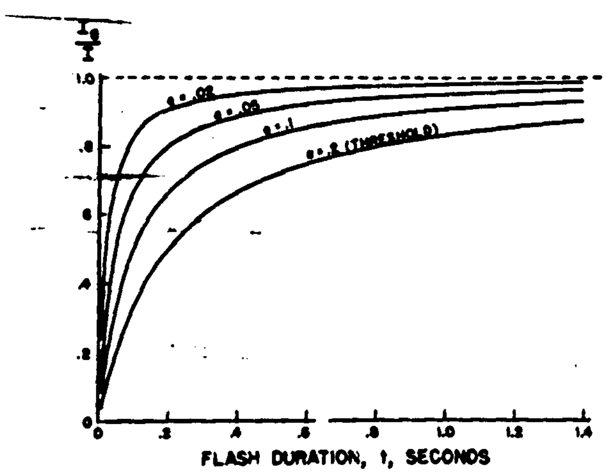


**Figure 2. Effective intensity ratio, $I_e/I$, as a function of flash duration, according to the Blondel-Rey equation (equation 1), for the case where a steady burning light of intensity $I$ is exposed for a time, $t$. Curves are shown for threshold illuminance when $a = 0.2$, and for above-threshold illuminances when $a$ has the values 0.1, 0.05, and 0.02.**

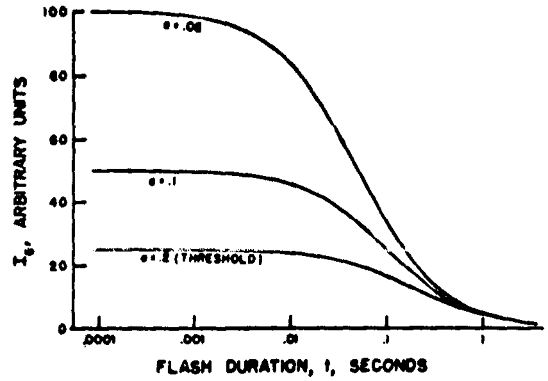


**Figure 3. Effective intensity, $I_e$, as a function of flash duration, $t$, for flashes of equal energy, at $a = 0.2$ (threshold illuminance), 0.1, and 0.05, plotted from equation 3, with $k$ arbitrarily chosen as equal to 5.**

wherein comparisons between flashing and steady lights or between flashing lights of very different flash durations do not give the same results when the comparisons are made close to the lights and illuminances are high as they do at considerable distances where the illuminances are very low.

For flashing lights of the second category, where there is no "waste" energy, a more meaningful presentation of the Blondel-Rey equation is shown in Fig. 3. In these curves the effective intensity for flashes of equal luminous energy is plotted against flash duration.

To obtain the curves of Fig. 3, the numerator of the right side of the Blondel-Rey equation, written in the form:

$$I_e = \frac{It}{a+t} \qquad (2)$$

is kept constant by varying $I$ and $t$ inversely with respect to each other so that

$$I_e = \frac{k}{a+t}, \qquad (3)$$

and plotting $I_e$ against $t$. $I_e$ becomes zero if $t$ is increased sufficiently. As $t$ is decreased, $I_e$ increases, and finally, when $t$ becomes very small compared to $a$, $I_e$ levels off to a value given by:

$$I_e = \frac{It}{a}.$$

At threshold, when $a = 0.2$,

$$I_e = 5\,(It).$$

As in the case of shutter flashing in the first category, the relative effectiveness of an above-threshold flash is greater than it is at threshold. For example when $a = 0.05$,

$$I_e = 20\,(It).$$

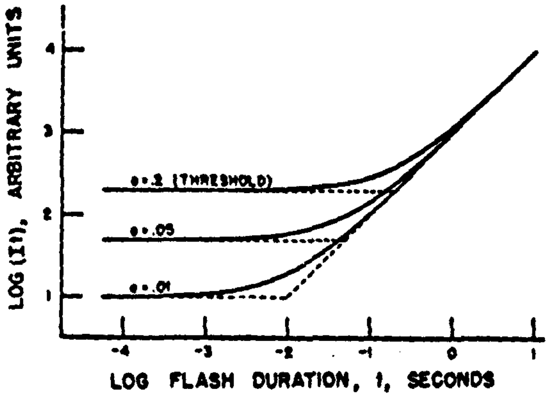


**Figure 4.** Flash energy (plotted as $It$) required to produce a given effective intensity, as a function of flash duration, at $a = 0.2$ (threshold illuminance), 0.05 and 0.01.

It will be noted that as $a$ is decreased, the duration time $t$ at which the curves level off is decreased, but in every case, leveling off occurs, and further reduction of the time of the flash produces no further increase in $I_e$ for flashes of constant energy. Thus for extremely short flashes, such as those obtained with electronic flash tubes ("strobe" lights), $I$ is reciprocally related to $t$ for a given $I_e$ according to the Blondel-Rey equation in a manner analogous to the equations used for determining photographic exposure.

The curves of Fig. 3 show the relationship between luminous energy in the flash and flash duration. A variation of this form of presentation, used by many research workers,[10,11] is shown in Fig. 4. In this figure, the luminous energy (plotted as $It$) necessary to produce a given effective intensity is plotted against flash duration. When the Blondel-Rey equation is plotted in this way, two distinct parts of the curves are clearly indicated. The portion at the left, for very short flash durations, shows the reciprocal relation between $I$ and $t$, as does Fig. 3, where

$$It = \text{constant}.$$

The part at the right indicates that for long durations

$$I = \text{constant}.$$

Between these two regions the curve blends smoothly from one limiting relation to the other. The two limiting relations are plotted as straight lines and their intersections, shown in the figure by dashed lines, define the "critical duration" or the duration marking the boundary time between the two relations. It may be noted that the critical duration at threshold is about 0.1 second. Fig. 4 may be compared with Fig. 1, from Blondel and Rey, which is similarly plotted except that the scales of Fig. 1 are linear rather than logarithmic. The logarithmic form of Fig. 4 shows clearly the two parts of the curve.

For a clear understanding of the significance of the curves of Fig. 4, and particularly to clarify the relationship of the curves for different values of $a$, it may be helpful to describe an experimental procedure for obtaining them. With observers stationed at some arbitrary distance from the light source, the intensity of the flash at very long duration is adjusted until the flashes appear to be at threshold. The curve for $a = 0.2$ is then obtained by reducing the flash duration by set amounts and determining the relative energy required to maintain the light at threshold. When that is done the light is readjusted to the intensity and duration prevailing at the beginning of the experiment and the observers are stationed at a point closer to the light by an amount sufficient to raise the illuminance to a level above threshold corresponding in this case to $a = 0.05$. A curve for $a = 0.05$ is then obtained as before (except that a comparison technique must be used instead of threshold observations) and this procedure is repeated for any additional values of $a$ desired.

### Integral Form of the Blondel-Rey Equation

Shortly after the publication of their classic paper on abrupt flashes, Blondel and Rey took up the question of flashes having time distributions other than square. On purely intuitive grounds they proposed the following modification of their original equation for effective intensity

$$I_e = \frac{\int_{t_1}^{t_2} I\,dt}{.21 + (t_2 - t_1)}. \qquad (4)$$

In this equation the numerator of the right side represents the light energy contained in the flash between the time limits of integration, $t_1$ and $t_2$. For the case where the flash is abrupt (square in shape) this reduces to the original equation

$$I_e = \frac{I(t_2 - t_1)}{.21 + (t_2 - t_1)} = \frac{It}{.21 + t}.$$

The difficulty with the integral equation proposed by Blondel and Rey is that there is an ambiguity about the choice of the time limits, $t_1$ and $t_2$. Blondel and Rey recognized this difficulty and could offer no rigorous solution for it. They did, however, suggest that $t_1$ and $t_2$ should be chosen as the times when $I$ was equal to the threshold value of $I$ for the case of steady illumination.

Recently Douglas* has proposed that the am-

*See Part II, paper by C. A. Douglas.

biguity in the selection of time limits in the integral form of the Blondel-Rey equation be resolved by selection of the limits so as to maximize $I_e$. When the limits of integration are so chosen, it turns out that the instantaneous value of intensity at these time limits is the steady light threshold intensity, exactly as proposed by Blondel and Rey in 1911. Douglas further shows that in general, for any value of $a$, the maximum value of $I_e$ is obtained when the limits of integration are those values of $t$ at which the instantaneous value of $I$ is equal to $I_e$.

Oddly, although most practical flashing light sources do not produce abrupt flashes, Blondel and Rey's proposal of the integral form of their equation has been generally ignored since 1911. Stiles, Bennett, and Green[4] took note of the proposal in 1937, but commented that there had been little or no experimental verification of it. Blondel and Rey[2] themselves, in 1916, made some measurements of effective intensity on rotating beacons with abrupt square form flashes, and with non-abrupt flashes of both long and short duration. The conditions of their experimental work were such as to give imprecise results, but within the error of their measurements, they confirmed the original equation and the integral form of it. As Douglas has shown, the uncertainty in the computation of $I_e$ resulting from the uncertainty in the choice of the limits of integration is not very large, especially for flashes of relatively short duration, so that experimental verification of the validity of the method of maximization of $I_e$ is very difficult to obtain. The uncertainty of Blondel and Rey's measurements was so great that the choice of limits of integration had no significant effect on their results.

Neeland, Laufer, and Schaub,[8] in 1938, reported the results of a series of measurements of effective intensity of rotating beacons, with both square and non-square flash characteristics. They used the results of their measurements to compute the value of $a$ in the Blondel-Rey equation, but fitted both the square and non-square flashes to equation (2). For the non-square flashes they took $I$ as the peak of the candlepower-time distribution and $t$ as the interval between those points on the distribution where the instantaneous intensity was 10 per cent of the peak intensity. They thus attributed much more energy to the flash than was actually there. The values of $a$ they obtained for square-form flashes conformed fairly well with those obtained by others, but the values of $a$ for the non-square flashes were significantly higher. If, however, their results are computed from the integral form of the equation, then the values of $a$ are in approximate agreement with those obtained by others.

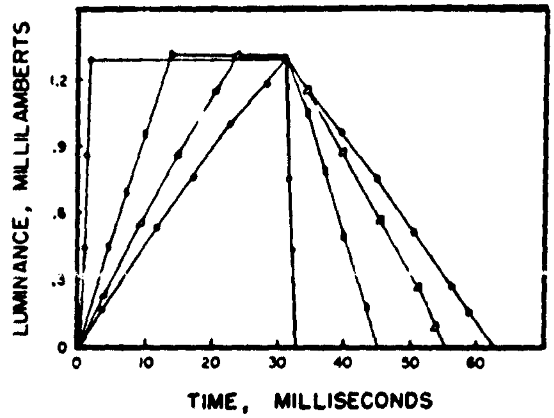


**Figure 5. Curves showing four of the seven wave forms used by Long in determining the relation between wave form of a flash and effective intensity. The wave forms are so designed as to provide equal total flash energies.**

In 1951, Long[10] reported an investigation of the relation between the wave form of the flash and the effective intensity. He used seven wave forms ranging from a square wave to a triangular wave. Fig. 5 shows four of the seven wave forms. The other three were intermediate in shape. The wave forms were so designed as to contain the same light energy per flash. The viewing condition was 15 degrees peripheral and the light source subtended a visual angle of 2 minutes. Long did not use the integral form of the Blondel-Rey equation nor did he concern himself with the choice of the limits of integration. He computed the total energy under the curve for each wave shape. Long felt that he was well within the range of reciprocity since his flash durations were quite short, and therefore looked only for the relation between the total

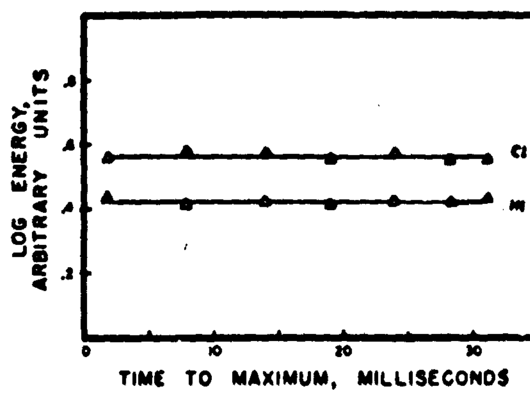


**Figure 6. Flash energy required for threshold excitation for flashes of seven wave forms (four of them shown in Fig. 5) by two observers. The seven wave forms are distinguished by the time required for the intensity to rise to a maximum. (Long).**

energy in the flash and the effective intensity. His results are shown in Fig. 6 (Long identified the different wave shapes by the time it took for the luminance to rise to a maximum.) It will be noted that the constancy of the energy required for threshold excitation with the different wave shapes was rather remarkable, in view of the experimental errors usually found in work of this kind.

It is of interest to consider Long's work from the point of view of Douglas' proposed method for computing $I_e$. For the case of the square form flash or the nearly square trapezoidal flashes, there is a negligibly small difference between $I_e$ computed from the total energy and $I_e$ maximized by Douglas' method. However, as the wave form becomes more nearly triangular, a larger difference may be expected. The computed difference for the triangular wave turns out to be about 10 per cent; that is, although the square form flash and the triangular form flash have equal total energy, $I_e$ for the triangular flash is 10 per cent less than $I_e$ for the square flash when both are computed from the integral form of the Blondel-Rey equation by Douglas' method of maximization. However, this difference amounts to only about 0.05 log unit on the ordinate scale of Fig. 6 and it is evident therefore that Long's results are not greatly affected by the change in the method of computing $I_e$.

### Extremely Short Flashes

As shown above, the relation between intensity and duration is reciprocal, according to the Blondel-Rey equation, for flashes with durations appreciably shorter than the "critical duration." It has been suggested, especially since the introduction of electronic flash lamps with flash durations of the order of fractions of a millisecond, that the reciprocity relationship does not hold for extremely short flashes, and that such flashes have higher effective intensity than that which would be predicted from the Blondel-Rey equation. On the other hand, some early research workers obtained results that suggested a failure of reciprocity in the other direction for short flashes, that is, the effective intensity was less than $I_e$ computed from the Blondel-Rey equation. Reeves,[6] for example, in 1918, found that it took about half again more energy in a flash at a duration of two milliseconds than in a flash of 10 milliseconds duration to obtain threshold illuminance. Piéron,[7,8] in 1920, found a similar reciprocity failure, although his results were less consistent. Stiles, Bennett, and Green,[4] in 1937, expressed doubt as to the validity of the inference of reciprocity drawn by Piéron and Reeves from their data. It appears likely, in view of the absence of any reciprocity failure in the results obtained in other work, most of it carried out with considerably more care, that the experimental error in the work of Piéron and Reeves was sufficiently great that an inference of reciprocity could have been drawn as validly as one of failure of reciprocity. Their papers furnished only sparse information as to experimental techniques, but it is likely for example that there may have been substantial errors in their measurements of the durations or energies of short flashes. Baumgardt,[9] in 1949, measured effective intensity in the duration range from four microseconds to one millisecond and found reciprocity throughout this range. His results are shown in Fig. 7.

The weight of the evidence for reciprocity is sufficiently great as to offer no support for the existence of a failure of reciprocity in either direction with any light sources now in use.

### Above Threshold Illuminances

Blondel and Rey observed during the course of their work on threshold measurements that a short duration flash that appeared equal in intensity to a long duration flash at threshold, appeared noticeably more intense than the long flash when the illuminance at the eye was well above threshold. They did not, however, pursue the question raised by this observation. Toulmin-Smith and Green,[12] in 1933, reported the results of a series of experiments with dark-adapted observers in which eye illuminances were varied from 0.2 mile-candle (slightly above threshold) to 4.0 mile-candles. In connection with signal lights used in navigation, the concept of "useful" threshold had been developed, as a measure of the lower limit of illuminance at which, under practical conditions, reliable recognition of the existence of light signals might be expected. It is generally agreed that this is in the neighborhood of 0.5 mile-candle, although under the variable conditions so often found in practice, wide departures from this value may be found. (Toulmin-Smith and Green used the rather over-precise value of 0.425 mile-candle as the "useful" threshold.) Their range of illuminances was therefore from about ½ to about 10 times the "useful" threshold, and bracketed fairly well the range of illuminances in which signal sightings usually occur. The flash durations were from 0.05 to 0.5 seconds. Fig. 8 shows the data obtained at an illuminance of 0.5 mile-candle and indicates the spread of the individual observations. Fig. 9 shows the family of curves obtained at the five illuminance levels tested. The Blondel-Rey equation for threshold illuminance is plotted in the figure and

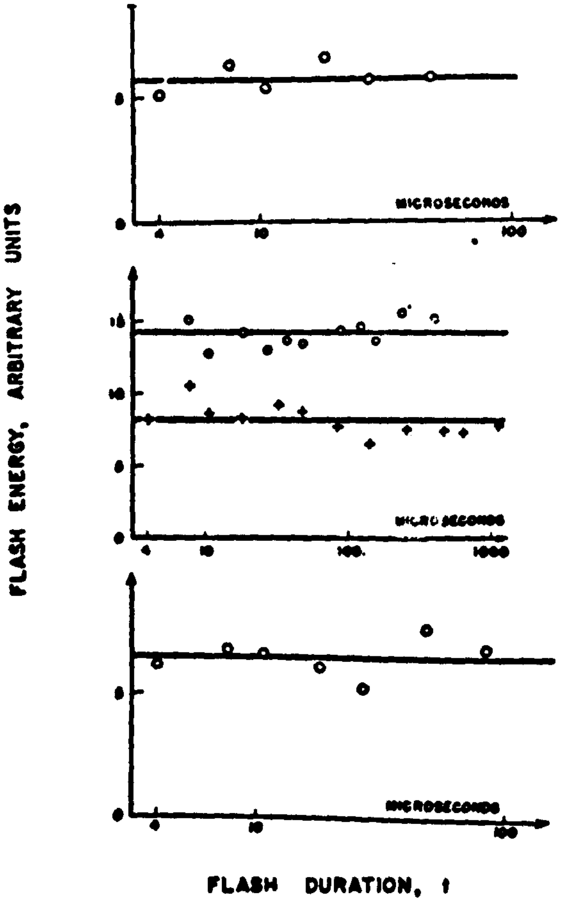


**Figure 7. Flash energy required for threshold excitation 12° from the fovea as a function of flash duration, *t*, for values of *t* between 4 microseconds and 1 millisecond. Data for four observers. (Baumgardt)**

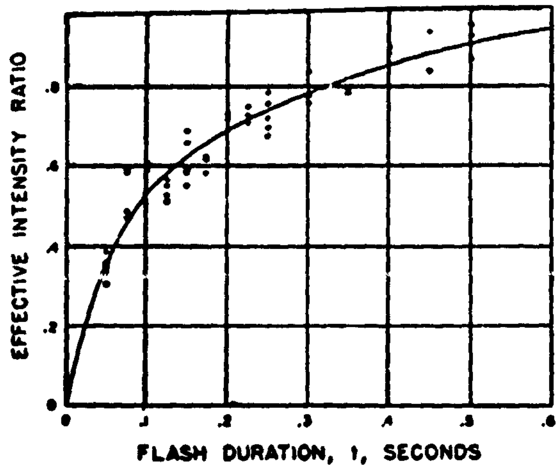


**Figure 8. Effective flash intensity relative to fixed intensity, as a function of flash duration, *t*, for each of two observers, at an illuminance of 0.5 mile-candle. (Toulmin-Smith and Green).**

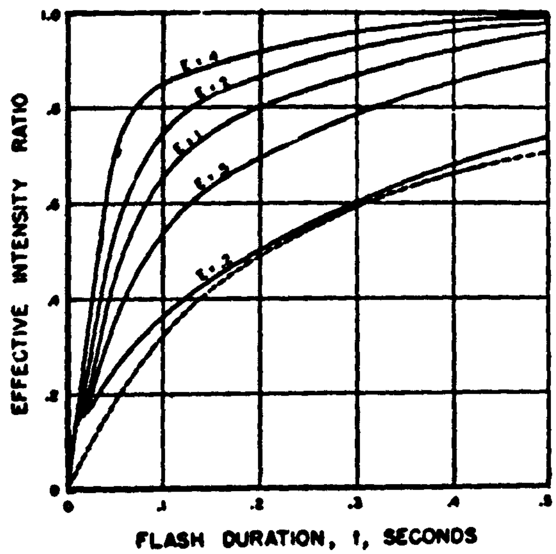


**Figure 9. Effective flash intensity relative to fixed intensity, as a function of flash duration, *t*, at illuminances $E$ = 0.2, 0.5 (data shown in Fig. 8), 1, 2, and 4 mile-candles. The dashed curve is the Blondel-Rey equation (equation 1) for threshold illuminance (Toulmin-Smith and Green).**

shows very close correspondence to the experimental curve at 0.2 mile-candle. Toulmin-Smith and Green tried to determine an equation of close fit to their data for the "useful" threshold at 0.425 mile-candle (obtained by interpolation from their experimental results) as shown in Fig. 10, in which are plotted the Blondel-Rey equation, interpolated test data for 0.425 mile-candle, and the fitted curve for the equation

$$\frac{I_e}{I} = 1.1\frac{t}{.15+t}. \qquad (5)$$

Hampton[14] in 1934 objected to the Toulmin-Smith and Green equation because of the coefficient 1.1, asserting that an adequate fit to all of their data could be obtained by an equation in the Blondel-Rey form with the constant, $a$, treated as a function of the illuminance level. On this assumption, he plotted Toulmin-Smith and Green's data as shown in Fig. 11, and from this obtained the equation,

$$\frac{I_e}{I} = \frac{t}{(.0255/E)^{.81}+t} \qquad (6)$$

where $E$, the illuminance at the observer, is given in mile-candles. In this equation, $a$ has the follow-

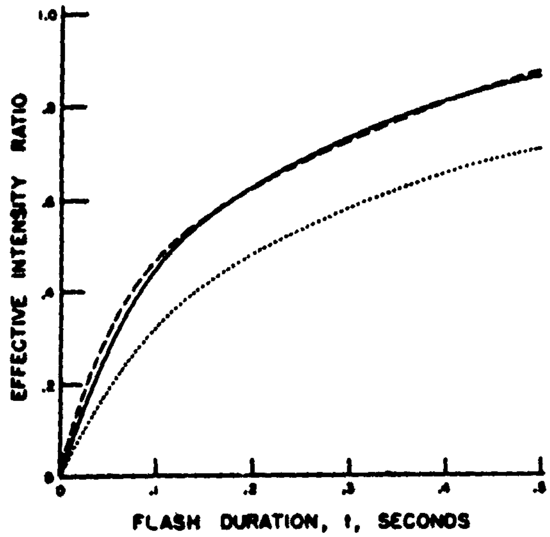


**Figure 10. Curve fitted to test data at 0.425 mile-candle ("useful" threshold) by Toulmin-Smith and Green. The dashed curve is the test data, obtained by interpolation from the data represented by the curves of Fig. 9. The solid curve is the fitted equation (equation 5). The lower dotted curve is the Blondel-Rey equation (equation 1) for threshold illuminance.**

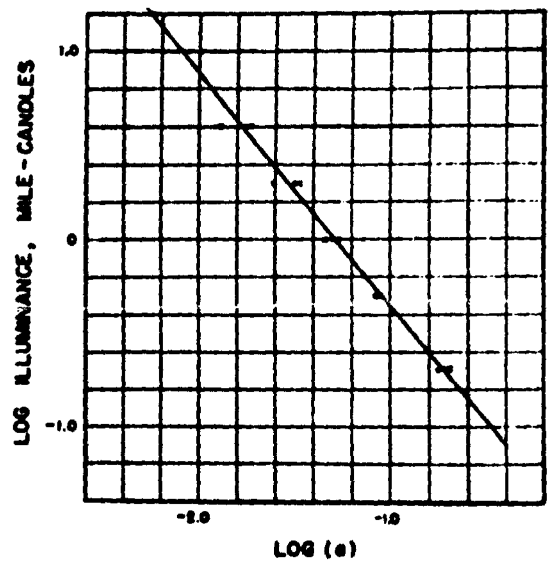


**Figure 11. Relation between the value of *a* and the illuminance, plotted by Hampton on the assumption that the Toulmin-Smith and Green data could be represented by the Blondel-Rey equation (equation 1).**

ing values at the illuminance levels used in the original experiments:

| Illuminance, Mile-Candles | *a* |
|---|---|
| .2 | .19 |
| .5 | .09 |
| 1.0 | .06 |
| 2.0 | .03 |
| 4.0 | .017 |

Hampton then replotted the original curves, using his equation, and obtained the curves of Fig. 12, not significantly different from those of Toulmin-Smith and Green (Fig. 9).

In Fig. 13, Hampton compared his equation (equation 6), Toulmin-Smith and Green's formula (equation 5), and the experimental data for 0.425 mile-candle. None of the differences is significant, in view of the experimental errors inherent in this type of measurement.

Apart from the work of Toulmin-Smith and Green, there appears to have been no systematic effort to ascertain the relationship between effective intensity and illuminances above threshold. Some qualitative observations and occasional rough measurements[1,2,12] have shown that effective intensity is relatively greater above threshold than at threshold, but have not confirmed with any precision the values of *a* calculated by Hampton from the data of Toulmin-Smith and Green. On the other hand, Schuil,[13] in the course of some work in which the value of *a* was incidental, noted that he did not confirm the Toulmin-Smith and Green value of *a* at an illuminance of 2.0 mile-candles although he did confirm it at 0.5 mile-candle. He did not give any details nor did he mention the extent of the disagreement. In the absence of any definitive work calling for a modification, it appears that

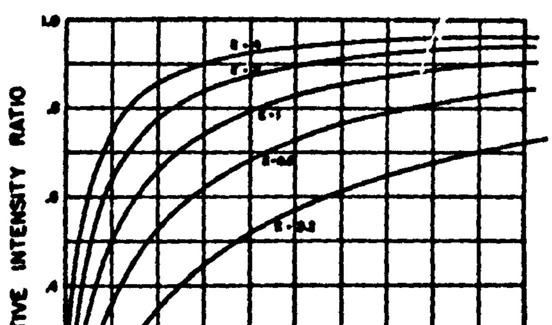


**Figure 12. Representation of Toulmin-Smith and Green data (Fig. 9) as recast by Hampton in the form of the Blondel-Rey equation, with the value of *a* empirically determined from the data (equation 6).**

Hampton's equation (equation 6) should be used in calculating effective intensity for above-threshold illuminances in the range covered by the observations of Toulmin-Smith and Green.

**Repeated Flashes**

The foregoing discussion of light flashes has considered a light flash as an isolated event in time, and the effective intensity of such isolated flashes is well described by the Blondel-Rey law. If flashes occur in more or less rapid succession, interaction effects may occur and it is of interest to know what these effects are. Many flashing signals used in practice do involve repetitive flashing signals, for example, aircraft "anti-collision" lights, traffic obstruction markers, lighthouse beacons, etc., so that the question is of some importance. For flashes above the critical flicker frequency, that is, at a repetition rate great enough so that the observer is unaware of the discontinuous character of the signal, Talbot's law holds, and the apparent intensity is equal to the average intensity. Under such conditions, it is immaterial how the luminous energy is distributed in time: two light sources with distributions whose time averages of intensity are equal will appear equally intense. Schuil[18] investigated this problem for flashes of $^1/_{10}$ and $^1/_{40}$ second duration, and with repetition rates from one per second up to frequencies well above the limit of noticeable flicker. In his experimental procedure he kept the flash duration constant (at either $^1/_{10}$ or $^1/_{40}$ second) and varied the duration of the dark interval between flashes. Thus the energy per flash was constant but the time average of the intensity increased as the dark interval decreased. Schuil's results, for an instantaneous illuminance during the flashes of 0.5 mile-candle, are shown in Fig. 14. Three regions, indicated in the figure for curve 2, the experimental results for a flash of $^1/_{40}$ second duration, may be distinguished. In the first, A, at the left, the light appears steady, and Talbot's law (curve 4 for the $^1/_{40}$ second flash) holds. This region begins at the point where the dark interval is zero and the total period of the flash and its duration are equal, the light therefore being physically continuous. The region ends at about .075 second when the repetition rate is about 13 flashes per second and the light-dark ratio is about 1:3. The effective intensity is then about one-third of the instantaneous intensity during the flash in accordance with Talbot's law. As the dark interval is increased further and the repetition rate correspondingly decreased (B), flicker is observed until the period is about 0.2 second long, corresponding to a repetition rate of five flashes per second. Beyond this point (C) the flashes are observed as more or less distinctly separate, and the effective intensity levels off to a value equal to that called for by the Blondel-Rey equation, with the value for $a$ computed by Hampton from the data of Toulmin-Smith and Green. The data show a smooth transition from the region at the left where Talbot's law holds to the region at the right where the effective intensity of the flashes is equal to that of isolated flashes in accordance with the Blondel-Rey law. Schuil observed, however, as noted previously, that while the general relationship between effective intensity and repetition rate which he obtained at 2.0 mile-candles was similar to that obtained at 0.5 mile-candle, the effective intensities for the limiting frequencies at the right differed from those of Toulmin-Smith and Green.

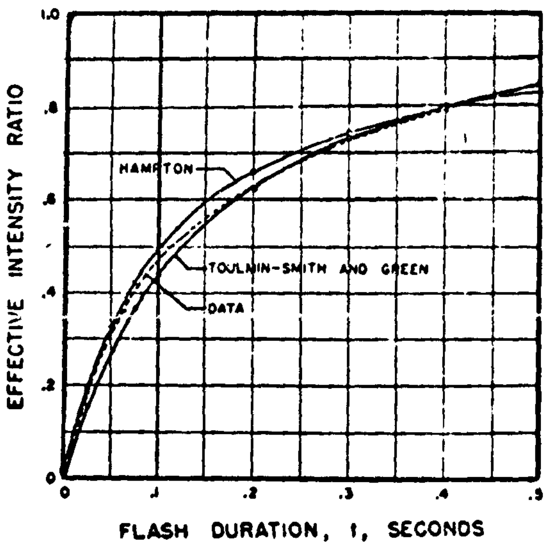


**Figure 13. Comparison of Toulmin-Smith and Green equation, Hampton equation, and data, at illuminance of 0.425 mile-candle. (Hampton)**

Although Schuil's work shows a smooth transition from the region of Talbot's law to the region of the Blondel-Rey law, some other work suggests that at certain middle frequencies in the flickering range, the effective intensity is actually greater if a steady burning light is occulted than if it is viewed constantly. Bartley[16] found for example that at a repetition rate of about 8 or 9 flashes per second, a flashing field of view was matched in apparent brightness with a similar steady field when the luminance of the flashing field was only about 60 per cent of the luminance of the steady field. This result was obtained with a steady field luminance of about 300 footlamberts and a light-dark ratio for the flashing field of 1:1. Bartley did not give

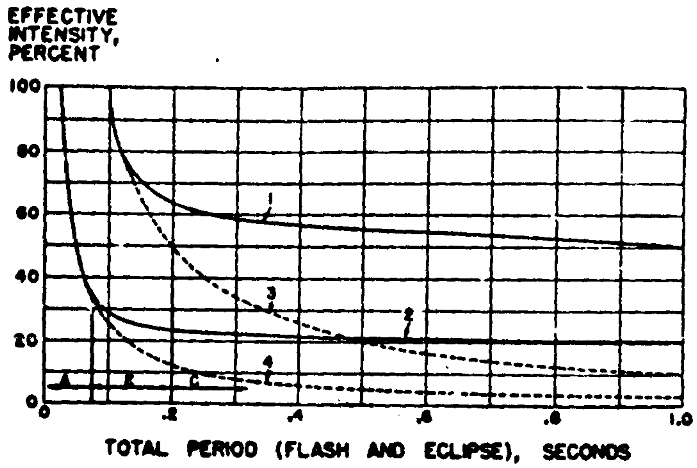


Figure 14. Effective intensity of flash relative to fixed intensity, for repeated flashes, as a function of the period of the flash cycle, with the flash duration kept constant and the eclipse time varied to obtain different repetition rates. (Schuil).

Curve 1, experimental curve for 1/10 second flash.

Curve 2, experimental curve for 1/40 second flash.

Curve 3, Talbot's law for 1/10 second flash.

Curve 4, Talbot's law for 1/40 second flash.

(*Note:* Three regions for curve 2 are distinguished: A, in which the light appears steady, B, in which the light appears to flicker, and C, in which the flashes are seen as separate and distinct.)

any data as to the size of the fields viewed, but it is presumed that the fields had some extent since he dealt with them in terms of luminance rather than intensity. Bartley gave data only for the results obtained with the 300-footlambert steady field, but noted that he obtained essentially similar results with steady field luminances from about 14 to about 1200 footlamberts. It will be noted that Bartley's minimum luminance was very far above threshold: 500 times threshold even for a light-adapted observer. Schuil, on the other hand, worked close to threshold. The author has carried out rough measurements which confirm the smooth transition found by Schuil in the low illuminance region. It appears likely that at illuminance levels where the concept of effective intensity is useful repetitive flashes that appear separate and distinct from each other may be treated in accordance with the Blondel-Rey law.

### Conclusions

Since it was first proposed in 1911, the Blondel and Rey equation

$$I_e = \frac{\int_{t_1}^{t_2} I\,dt}{a + (t_2 - t_1)}$$

defining the effective intensity of a flashing light in terms of the duration of the flash and the instantaneous intensity during the flash has become well established. A considerable body of research has verified the general validity of the equation over a large range of observing conditions. There are nevertheless many areas where the precise application of the equation is in some doubt and where considerably more work is called for in order to establish valid relationships or to confirm relationships at present only poorly supported.

For point sources of non-chromatic light, foveally viewed at threshold with the dark-adapted eye, Blondel and Rey's value of about 0.2 for the constant, *a*, seems fairly well established, although somewhat higher and lower values have been reported, ranging from about 0.15 to 0.25. The inherent lack of precision of measurements in this field suggests that the value of 0.2 be used in all computations as representative of the values most often found. Observations of flashing lights not falling within the narrow confines set forth above, for example, lights that are chromatic, or with some extent, or viewed peripherally, seem to be subject to more or less the same relationship as long as effective intensity is defined in terms of the same source burned steadily and viewed under the same conditions.

The general effect of raising the illuminance at the eye above threshold or of providing a light background against which the lights are viewed is to lower the value of *a* in the Blondel-Rey equation. For the case of above-threshold viewing of flashing lights against a dark background, values of *a* related to illuminance at the eye have been established, but considerably more work could be done in this area to put these values on a firmer foundation. Because of the critical dependence of the threshold illuminance on the conditions of observation, it may be profitable in research work to relate the results to the steady light threshold for the same conditions of observation.

Finally it seems clear that the concept of effective intensity should be limited in application to low levels of illuminance, with the illuminance here considered as a multiple of the marginal amount of illuminance necessary to distinguish a flashing light from its background.

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# Computation of the Effective Intensity Of Flashing Lights

By CHARLES A. DOUGLAS

It is generally recognized that when a light signal consists of separate flashes, the instantaneous intensity during the flashes must be greater than the intensity of a steady light in order to obtain threshold visibility. Blondel and Rey[1] found that the threshold illuminance for an abrupt flash (a flash producing a relatively constant illuminance throughout its duration) is

$$E = E_o(a+t)/t, \quad (1)$$

where $E_o$ is the threshold illuminance for a steady light, $t$ is the flash duration, and $a$ is a constant. They found that $a$ was equal to 0.21 when $t$ is in seconds.

It is convenient to evaluate flashing lights in terms of their effective intensity, $I_e$, that is, the intensity of a fixed light which will appear equally bright. Then

$$I_e = I E_o/E$$

where $I$ is the instantaneous intensity producing the illuminance, $E$,

$$I_e = \frac{It}{a+t}. \quad (2)$$

Later Toulmin-Smith and Green[2] found that somewhat different effective intensities were obtained when the illuminance at the eye was above threshold. However, Hampton[3] showed that their experimental results could be adequately expressed by equation (2) when $a$ is a function of the illuminance at the eye.

The flash from most lights used in aviation service, such as airway beacons and anti-collision lights, is not abrupt. The instantaneous intensity often rises and falls gradually and may vary appreciably during the flash. If the flash duration is very short or if the times of rise and fall of intensity are short in comparison to the flash duration, only small uncertainties would be introduced in the determination of flash duration and by the use of the product of the peak intensity during the flash and the flash duration for the quantity $It$. However, in many cases significant errors would be introduced. Some modification of equation (2) is therefore required.

Some of the specifications for flashing lights have evaluated their signals in terms of the candle-seconds in the flash, integrating over a period of not more than 0.5 second, that is

$$\text{Candle-seconds} = \int_{t_1}^{t_2} I\,dt$$

where $I$ is the instantaneous intensity and $t_2 - t_1$ does not exceed 0.5 second. This method of evaluation provides a measure of comparison between lights of roughly the same intensity variation with time but is not suited to the comparison of lights of different flash characteristics nor to the computation of visual ranges.

Others have used the relation

$$I_e = \frac{I_{max}t}{a+t},$$

where $I_{max}$ is the maximum instantaneous intensity during the flash and $t$ is the flash duration. Often the value of $a$ is adjusted for the characteristics of the flash so that the computed value of $I_e$ is in reasonable agreement with the observed value.

When the specification for aircraft anti-collision lights was being drafted, it was suggested that a modified form of equation (2) be used for the computation of effective intensity, so that

$$I_e = \frac{\int_{t_1}^{t_2} I\,dt}{.2+t_2-t_1}. \quad (3)$$

An equation of this form was originally suggested by Blondel and Rey,[4] but has rarely been used.

The question of choice of limits was immediately raised. Rather than use an arbitrary set of limits, such as choosing for $t_1$ and $t_2$ the times when $I$ was 10 per cent of the peak of the flash, a choice of limits which would make $I_e$ a maximum was suggested. This immediately poses the problem of developing a method, other than trial and error, of obtaining the maximum value of $I_e$. The development of such a method is the purpose of this paper.

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**Fundamental Theorems**

The method of obtaining the maximum value of $I_e$ will be developed by means of one theorem and two corollaries. The proofs follow.

Theorem. *$I_e$ is a maximum when the limits $t_1$ and $t_2$ are the times when the instantaneous intensity is equal to $I_e$.*

This theorem may be readily proved by application of the calculus of variations.* The proof given below is included because of the application of the method and equations used to later sections of the paper.

Consider instantaneous intensity, $I$, in a flash as any continuous, non-negative, single-valued function of time such that $I$ is less than $I_e$ in the intervals $t''_1$ to $t_1$ and $t_2$ to $t''_2$, and $I$ is greater than $I_e$ in the intervals $t_1$ to $t'_1$ and $t'_2$ to $t_2$, where $t''_1 < t_1 < t'_1 < t'_2 < t_2 < t''_2$, and $I_e$ is defined in equation (4):

$$I_e = \frac{\int_{t_1}^{t_2} I\,dt}{a + t_2 - t_1}. \qquad (4)$$

Fig. 1 shows $I$ as a function of $t$ for a simple one-peak flash meeting these requirements.

*Case I.*

Consider the case where the integration is performed over the time interval $t'_1$ to $t'_2$ which lies within the interval $t_1$ to $t_2$.

Then the intensity $I'$ at the times $t'_1$ and $t'_2$ is greater than $I_e$.

$$I' > I_e.$$

Let

$$I'_e = \frac{\int_{t'_1}^{t'_2} I\,dt}{a + t'_2 - t'_1}.$$

Then

$$\int_{t_1}^{t_2} I\,dt = \int_{t_1}^{t'_1} I\,dt + \int_{t'_1}^{t'_2} I\,dt + \int_{t'_2}^{t_2} I\,dt,$$

so that

$$I_e(a + t_2 - t_1) = \int_{t_1}^{t'_1} I\,dt + I'_e(a + t'_2 - t'_1) + \int_{t'_2}^{t_2} I\,dt.$$

But

$$\int_{t_1}^{t'_1} I\,dt > I_e(t'_1 - t_1), \qquad (5a)$$

and

$$\int_{t'_2}^{t_2} I\,dt > I_e(t_2 - t'_2). \qquad (5b)$$

Substituting and combining terms, we have

$$I_e(a + t'_2 - t'_1) > I'_e(a + t'_2 - t'_1).$$

Therefore

$$I_e > I'_e. \qquad (6)$$

*The author is indebted to Dr. H. H. Seliger of the National Bureau of Standards for an elegant proof using the calculus of variations.

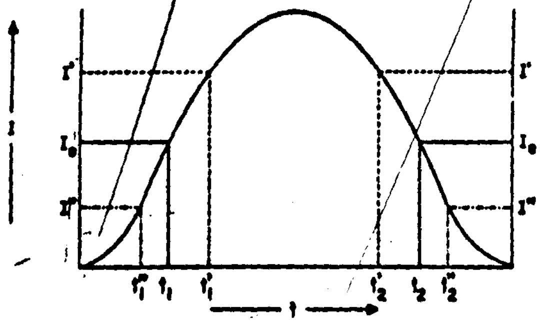


**Figure 1. Intensity-time distribution of simple flash. (See text.)**

*Case II.*

Consider now the case where the integration is performed over the time interval $t''_1$ to $t''_2$, which includes the interval $t_1$ to $t_2$.

Then the intensity $I''$ at the times $t''_1$ and $t''_2$ is less than $I_e$.

$$I'' < I_e.$$

Let

$$I''_e = \frac{\int_{t''_1}^{t''_2} I\,dt}{a + t''_2 - t''_1}.$$

Then

$$\int_{t''_1}^{t''_2} I\,dt = \int_{t''_1}^{t_1} I\,dt + \int_{t_1}^{t_2} I\,dt + \int_{t_2}^{t''_2} I\,dt,$$

and

$$I''_e(a + t''_2 - t''_1) = \int_{t''_1}^{t_1} I\,dt + I_e(a + t_2 - t_1) + \int_{t_2}^{t''_2} I\,dt. \qquad (7)$$

But

$$\int_{t''_1}^{t_1} I\,dt < I_e(t_1 - t''_1), \qquad (8a)$$

and

$$\int_{t_2}^{t''_2} I\,dt < I_e(t''_2 - t_2). \qquad (8b)$$

Substituting and combining terms, we have

$$I_e > I''_e. \qquad (9)$$

Thus $I_e$ is greater than both $I'_e$ and $I''_e$. Therefore, the maximum value which can be obtained from the Blondel-Rey relation equation (4), is that obtained when the intensity at the beginning and end of the interval of integration is equal to the effective intensity.

Corollary 1. *If the instantaneous intensity is integrated over a period of time $t'_1$ to $t'_2$ shorter than $t_1$ to $t_2$, and $I'$ is the instantaneous intensity at these times, a value $I'_e$ is obtained for the effective intensity that is always less than $I'$.*

From equation (6) we have

$$I_e > I'_e.$$

But

$$I' > I_e.$$

Therefore

$$I' > I'_e. \quad (10)$$

Corollary 2. *If the instantaneous intensity is integrated over a period of time $t''_1$ to $t''_2$ longer than $t_1$ to $t_2$, and $I''$ is the instantaneous intensity at the times $t''_1$ and $t''_2$, a value $I''_e$ is obtained for the effective intensity that is always greater than $I''$.*

From equation (7) we have

$$I''_e(a+t''_2-t''_1)=\int_{t''_1}^{t_1} I\,dt + I_e(a+t_2-t_1)+\int_{t_2}^{t''_2} I\,dt.$$

But

$$\int_{t''_1}^{t_1} I\,dt > I''(t_1-t''_1),$$

and

$$\int_{t_2}^{t''_2} I\,dt > I''(t''_2-t_2).$$

Also

$$I_e(a+t_2-t_1) > I''_e(a+t_2-t_1).$$

Substituting these into equation (7) and simplifying, we have

$$I''_e > I''. \quad (11)$$

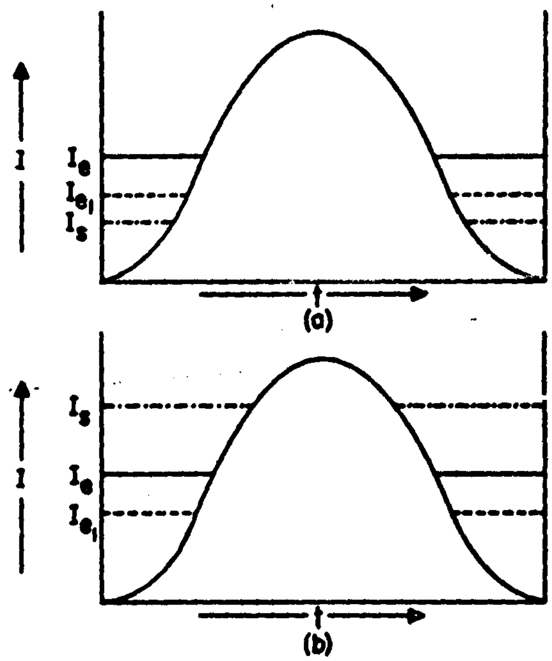


**Figure 2. Illustrating the method of determining conformance to specification requirements of effective intensity. (See text.)**

## Computations of Effective Intensity

Guides for the computation of the effective intensity from an intensity-time distribution curve may be obtained from the theorem and corollaries.

### *a. Computation of $I_e$*

1. Make an estimate $I'$ of the value of the effective intensity and solve equation (3) using the values of $t$ corresponding to this intensity, obtaining $I_{e_1}$.

2. Repeat step 1 above, using as limits the values of $t$ corresponding to the $I_{e_1}$ obtained in step 1, obtaining $I_{e_2}$. Repeat as often as necessary to obtain the desired accuracy.

Note that if the estimated effective intensity is too high ($I'$ in Fig. 1), the effective intensity, $I_{e_1}$, computed in step 1 will be below $I_e$ ($I''$ of Fig. 1) and thus $I_e$ lies between $I'$ and $I_{e_1}$. If the initial estimate is lower than $I_e$ ($I''$ of Fig. 1), $I_e$ will be greater than both $I''$ and $I_{e_1}$ and a "straddle" is not obtained but $I_e$ is approached continuously from the low side.

### *b. Determination of Conformance of a Flashing Light to Specification Requirements*

1. Compute $I_{e_1}$ using the time limits corresponding to the specified effective intensity $I_s$. If $I_{e_1}$ is greater than $I_s$, the unit obviously complies, for the conditions are those of Fig. 2a (corollary 2).

2. If $I_{e_1}$ is equal to $I_s$, the unit just complies, for then $I_s = I_e = I_{e_1}$ (theorem).

3. If $I_{e_1}$ is less than $I_s$, the unit fails for then the conditions are those of Fig. 2b (corollary 1).

Note that the degree by which the unit exceeds or fails to meet the specification requirements is *not* given by the single computation described here. The method outlined in section *a* must be used for this purpose.

### *c. Visual Range Computations*

If the visual range of the light, under specified conditions of transmittance and threshold, is desired, compute the effective intensity by using the method outlined in section *a* and compute the visual range by using Allard's Law.

If the problem is only the determination of whether the light can be seen at a given distance under specified conditions of transmittance and threshold, use Allard's Law to compute the fixed intensity required to make the source visible at this distance. Then, by using the method outlined in section *b*, determine if the effective intensity of the unit exceeds this intensity.

## Application to Complex Intensity-Time Curves

Not all units have single-peak intensity-time distribution curves similar to the curve shown in Fig. 1. Consider an intensity-time distribution curve of the type whose rise is shown in Fig. 3 where $I_s$

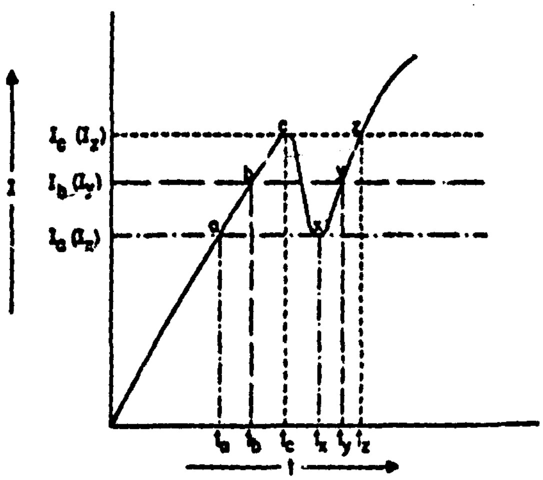


**Figure 3. Illustrating application of the method to multi-peak intensity-time distributions. (See text.)**

is the average intensity in the time interval $t_b$ to $t_y$. (The time interval $t_y - t_x$ is sufficiently short so that the momentary decrease in intensity is not visible.) If $I_e$ is less than $I_a$ or is greater than $I_c$, then the restrictions on the shape of the curve stated in theorem 1 are met and there is no problem in the determination of $I_e$.

Consider the case where $I_e$ lies between $I_a$ and $I_c$. It may be easily shown by means of equations (5) and (8) that as the shape of other parts of the intensity-time distribution curve changes, the lower limits of time to be used to obtain the maximum value of equation (4) will lie between $t_a$ and $t_b$ or between $t_y$ and $t_z$ and will never lie between $t_b$ and $t_y$. If $I_e$ is equal to $I_b$, then either $t_b$ or $t_y$ can be used as the lower limit.

## Application to Groups of Short Flashes

In general a signal from a flashing light consists of regularly spaced single flashes of light and the

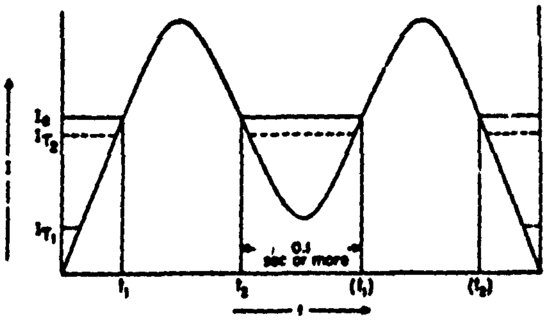


**Figure 4. Illustrating application of the method to two-peak intensity-time curves. (See text.)**

interval between flashes is so great that each flash has little influence on the effective intensity of the adjoining flashes.

Consider first a flash with the intensity-time distribution shown in Fig. 4. This flash is similar to that of the "split-beam" beacon used at military airfields. If the threshold intensity required to make a steady light visible is much less than $I_e$ $(I_{T_1})$, the flash will be seen as a continuous flash with two peaks. However, if the threshold intensity is about equal to $I_e$ $(I_{T_2})$, two separate flashes will be seen. The maximum distance at which the light can be seen will be determined by the effective intensity of a single flash computed over the time interval $t_1$ to $t_2$.

There are lights that produce a number of very short flashes in rapid succession so that this group of flashes is seen as a single flash. An example of a light of this type is a unit using a number of condenser-discharge lamps to produce a single flash.

There appear to be no published data reporting studies of the effects of groups of flashes where the interval between flashes is short. Behavior of the eye under somewhat similar conditions suggests that if in a group of flashes the periods during which the instantaneous intensity of the light is below the effective intensity of the flash are of the order of 0.01 second or less, the eye will perceive this group as a single flash. The effective intensity of the group should then be computed by equation (12), choosing as times $t_1$ and $t_2$ the first and the last times the instantaneous intensity is $I_e$.

$$I_e = \frac{\int_{t_1}^{t_a} I\,dt + \int_{t_b}^{t_c} I\,dt + \int_{t_d}^{t_e} I\,dt + \int_{t_f}^{t_2} I\,dt}{a + t_2 - t_1}. \qquad (12)$$

Note that $I_e$ is the effective intensity of the group and not that of a single flash.

If the periods during which the effective intensity is less than $I_e$ are of the order of 0.1 second or more, it is believed that the individual flashes will be seen. Therefore, the effective intensity should then be computed on the basis of a single flash.

When the dark period is between 0.01 and 0.1 second, the effective intensity will lie between that of a single flash and that of the group. The behavior during the transition is not known.

## Numerical Examples

Although the precise determination of the maximum value of $I_e$ may appear laborious, it is relatively easy. Since a change in the times chosen as the limits in equation (4) changes the denominator and the numerator in the same direction, it is not necessary to determine the correct limits, $t_1$ and $t_2$,

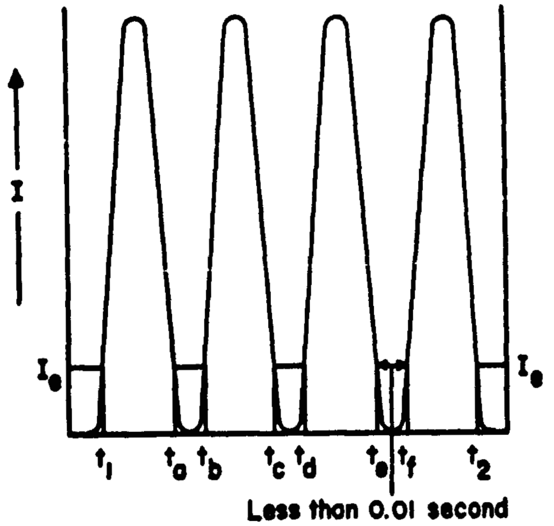


**Figure 5. Illustrating application of the method to multiple flashes.**

with great precision in order to obtain a satisfactorily precise determination of $I_e$. This is illustrated in Fig. 6 which shows two representative intensity-time distributions, one for a flashing light with a flash duration of about one-quarter second, and one with a flash duration of about one-twentieth second. The values of $I_e$ are computed for seven sets of time limits. The values obtained are indicated at the abscissa of the time limits. The middle value of each group is the maximum $I_e$ computed according to the method outlined above. Note that this value is equal to the instantaneous intensity at the corresponding time limits. Typically, the maximum effective intensity occurs lower on the curve for the short duration flash than for the long one and the variation of computed values of effective intensity with changes in time limits is smaller.

Experience indicates that if the times chosen for the initial integration are the times when the instantaneous intensity is about 20 per cent of the peak intensity, only one additional step is required to obtain a value for the effective intensity which is within one or two per cent of the maximum value. This is within the limits of accuracy with which the integral is evaluated by means of a planimeter. Often a single computation is sufficient if, instead of using as limits for the initial integration the times when $I_e$ is 20 per cent of the peak intensity, the times used are the times when the instantaneous intensity is equal to the product of the peak intensity and the number of seconds between the times when the instantaneous intensity is roughly five per cent of the peak intensity.

The maximum value of $I_e$ for the curves of Fig. 6 were computed by using as limits for the first integration the times corresponding to an intensity equal to about 20 per cent of the peak intensity and as limits for each succeeding integration the times corresponding to the instantaneous intensity obtained from the preceding step. (The curves have the shape of probability curves so the values of integrals may be computed with the desired accuracy. The accuracy is not limited by the accuracy of planimetric measurements.) Successive values for $I_e$ of 1.66, 1.75, and 1.75 kilocandles were obtained for the longer flash and of 3.41, 3.42, and 3.42 kilocandles for the shorter flash.

## Discussion

As noted above, concern has frequently been expressed about the choice of the limits for the integral of the Blondel-Rey relation for computing the visual range of a flashing light. It seems illogical to extend the limits of the integral beyond the

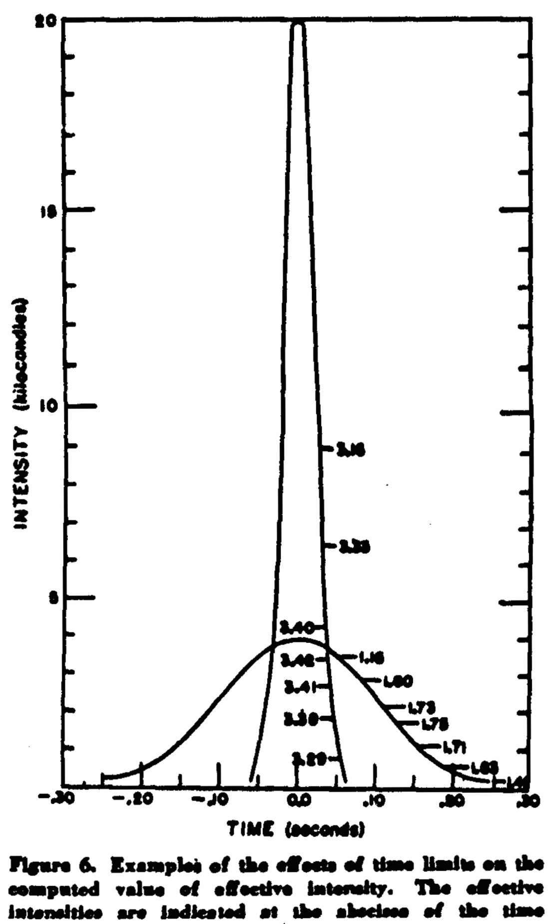


**Figure 6. Examples of the effects of time limits on the computed value of effective intensity. The effective intensities are indicated at the abscissa of the time limits used in the computations.**

times when the instantaneous intensity is below the threshold intensity for steady burning lights so that intensities which are below threshold, even for a steady burning light, are included, or to exclude intensities which are above threshold for steady burning lights. Using this reasoning, Blondel and Rey[4] suggested that the limits of the integral of equation (4) be the times when the instantaneous intensity is equal to the threshold intensity. As shown above, these are also the limits which make the computed visual range of the light a maximum. Therefore, the use of these limits in evaluating the performance of a lighting unit appears to be a logical choice.

The use of the maximum value of $I_e$ as the effective intensity of a flashing light is probably not valid except when the light is at or near threshold. When the light is well above threshold, not only will the value of $a$ in equation (4) be decreased, thus tending to increase the value of $I_e$, but also the limits of the integral should probably be extended to include the entire portion of the flash which is above threshold, thereby tending to decrease the value of $I_e$. In many cases this latter effect will be predominant. This is consistent with the decrease in effective intensity of airway beacons with increase in illuminance at the eye found by Neeland, Laufer, and Schaub.[5]

This analysis should be considered only as a mathematical treatment of equation (4). The analysis neither proves nor disproves the validity of this equation in determining the effective intensity of flashing lights nor the validity of the principle of choosing the limits of integration so that the effective intensity is a maximum.

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2. Toulmin-Smith, A. K., and Green, H. N.: "The Fixed Equivalent of Flashing Lights," *Illuminating Engineer*, London, *XXVI*, 305 (1933).
3. Hampton, W. M.: "The Fixed Equivalent of Flashing Lights," *Illuminating Engineer*, London, *XXVII*, 46 (1934).
4. Blondel, A., and Rey, J.: "Application aux Signaux de la Loi de Perception des Lumières Brèves a la Limite de Leur Portée," *Journal de Physique*, *1* (5th Series), 643 (1911).
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## DISCUSSION

F. E. CARLSON:* These interesting papers should prove very helpful in evaluating the many lighting problems involving flashing lights. It is assumed that the calculations described hold for viewing conditions approximating the ideal. I wonder if the authors could tell us something about the influence of such factors as atmospheric transmission, other sources of light in the field of view, etc.

*General Electric Co., Large Lamp Dept., Cleveland, Ohio.

G. A. HORTON:* Messrs. Projector and Douglas are to be highly commended for presenting two papers on a subject which has been long neglected. The papers are particularly timely because of recent application of condenser discharge lights as aviation runway approach lights.

In the condenser light, the length of the flash may be of the order of 100 microseconds, while the peak of the flash may reach several million candlepower. Since the time required for the eye to reach its maximum response is of the order of one-tenth of a second, it is clear that the effective candlepower as seen by the eye is not the same as the measurable average candlepower developed by the flash.

However, by application of the formulas developed by the authors of these two papers to the measured data can the *effective candlepower of the condenser lights be expressed.*

I should like to ask the authors if visual field checks of the condenser lights have been made to substantiate the theory developed in the papers and, if so, what degree of agreement was obtained.

T. H. PROJECTOR and C. A. DOUGLAS:** It is a pleasure to acknowledge the comments of Messrs. Carlson and Horton and to reply to their questions.

Mr. Carlson raises the question of the relation between effective intensity and the conditions of observation. The term "effective intensity" is used for convenience, and implies narrowly restricted conditions, usually threshold illuminance, dark adaptation, etc. Strictly, the term should be "effective illuminance" for greater generality, thus automatically taking into account the effect of atmospheric attenuation on illuminance. The effect of other sources of light in the field of view and, in general, of the background conditions, state of adaptation, etc., has not been sufficiently explored and is a fertile field for further investigation.

In reply to Mr. Horton, there have been a number of visual field checks of the applicability of the Blondel-Rey law to condenser discharge lights data in the United States and in Europe. All of the results of these checks show no significant deviations from the Blondel-Rey law. To our knowledge none of these results has as yet been published formally. Publication of the results of work at the National Bureau of Standards is expected in the future. It should be noted that these remarks apply only to the direct light from the source when the illuminance from the light is near threshold. Frequently, in a foggy or hazy atmosphere, the visual range of the glow of a flashing light will be considerably greater than the visual range of the direct light.

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# Photometer for Luminescent Materials

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(Received March 12, 1945)

In evaluating the usefulness of luminescent materials it is necessary to take into account the behavior of the human eye at low values of luminance. A photometer that provides for the determination of low luminances, with due regard for the characteristic behavior of the eye at such values, is described. It is interesting to note that both the luminescent materials and some of the phenomena of vision for the nearly dark-adapted eye have been known for many years, although the use of modern lamps to produce higher and higher illuminations has made it generally unnecessary to consider these phenomena. However, the use of the airplane for bombing with the counter measure of blacking out as a means of passive defense and the need for markers in the interiors of blacked-out ships have shown many of the luminescent materials to be practical instead of merely novel, and has led to development of methods for measuring the luminances which they yield.

## I. INTRODUCTION

SOME of the luminescent materials have been known for hundreds of years but were considered more or less as novelties until recently, when their practical usefulness became apparent. Luminescence not outlasting the excitation (fluorescence) was, in all probability, not observed until after the invention of irradiating sources and devices especially designed for the purpose of detecting such luminescence. Luminescence such as that of glowworms and rotting wood and that which outlasts the excitation (phosphorescence) must have been noticed by man in his early existence. Aristotle's pupil, Theophrastus, says that a carbuncle exposed to sunlight glows like a live coal;[1] while Aristotle, himself (about 350 B.C.) mentions the luminescence of rotting wood. Benvenuto Cellini tells of seeing a white sapphire which illuminated a perfectly dark room.[2] As early as 1652 Peter Poterius made little toy animals from phosphorescent material.[1] An alchemist, Vicenzo Cascariolo, in Bologna, Italy, about 1600 found a stone which seemed heavier than one of its size should be; upon heating it, in the hope of finding gold, he discovered that it would glow in the dark, "sometimes for as long as an hour" and also found how to make it glow at will.[3]

The luminance[4] of the luminescent light from phosphorescent materials ranges downward from that of a white surface viewed in full moonlight. In measuring such luminances it is necessary to

[1] P. Pringsheim and M. Vogel, *Luminescence of Liquids and Solids and its Practical Applications* (Interscience Publishers, Inc., New York, 1943).

[2] S. H. Ball, Sci. Monthly 47, 497 (1938).

[3] G. T. Schmidling, *Protective and Decorative Coatings* (John Wiley and Sons, Inc., New York), Vol. III, p. 657. Also reference 1.

[4] OSA Committee on Colorimetry, "Psychophysics of color," J. Opt. Soc. Am. 34, 245 (1944).

consider the behavior of the weakly illuminated eye. In 1825 J. Purkinje[8] showed that after a red surface and a blue surface have been illuminated so as to have the same brightness, a reduction of the illumination of both surfaces in the same proportion will cause the red surface to appear darker than the blue after a certain limit of reduction has been passed. The dependence of the observed values also upon the size of the field of view for the nearly dark-adapted eye was determined by P. Reeves[9] in 1917. The trend to higher and higher illumination for lighting purposes had made it unnecessary for the photometrist to remember these effects in the measurements customarily made in the laboratory. However, as the use of luminescent materials was shown to be practical it became necessary to take account of both of these phenomena in constructing a photometer for the measurement of their luminance.

## II. PHOTOPIC, MESOPIC, AND SCOTOPIC VISION

The human eye has the ability to adapt for light conditions over a wide range. The approximate upper limit is represented by the condition of viewing fresh snow in full sunlight, which is uncomfortable and, if long continued, results in temporary blindness, "snow blindness," or if

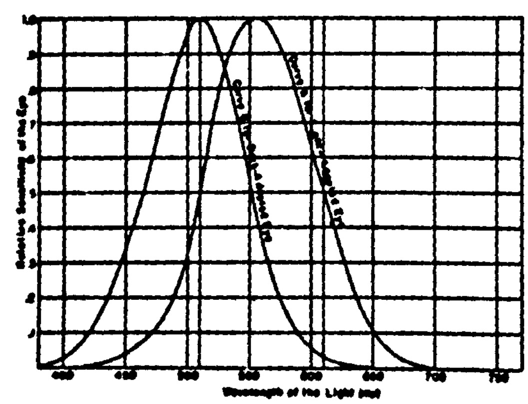

FIG. 1.

viewed for prolonged periods, may result in permanent injury to the eye. The lower limit is considerably below the condition of viewing a white surface on a clear, moonless night. The change in size of the pupil of the eye is easily observed; and, as everyone knows, the pupil is small if strongly illuminated, and as the illumination weakens, the pupil becomes larger. There are other less obvious changes, such as the change in the luminosity curve, which explains the Purkinje effect and the loss of the ability to detect detail (acuity) and of the ability to detect chromaticity differences, all associated with the transition from cone to rod vision.

When we view a surface of high brightness with photopic vision, that is, when the eye may be said to be light-adapted, we find that the eye has a nearly constant luminosity curve, independent of the luminance range under consideration. All definitions of units of luminance (photometric brightness) imply that comparisons between differently-colored surfaces be made at values of luminance sufficiently high to insure that the observer's eye is in a state of light adaptation. This is to insure that luminance values obey the additive law, by which a luminance of $y$ units superimposed upon one of $x$ units will provide a luminance of $(x+y)$ units. If this law is not obeyed, the ordinary inverse square law, the ratio of areas of openings in diaphragms, and the ratio of the areas of the open and opaque sectors of a rotating disk (Talbot's law) may not be applied to the luminance values under consideration. If differently-colored surfaces are viewed, obedience to the above-mentioned laws occurs only if the luminosity curve of the eye is constant and independent of the adaptive state of the eye throughout the range of luminance under consideration. Fortunately the eye in observing luminances greater than 1000 microlamberts does possess a nearly constant luminosity curve, and for this condition (when the eye is said to be light-adapted) we have photopic vision and the values of luminance (photometric brightness) for differently-colored surfaces not only obey the additive law but also correlate well with brightness, subjectively evaluated. We may speak of such values as photopic luminance.

Unfortunately the luminosity curve of the eye does not remain constant when the luminance of an extended surface is reduced below 1000 microlamberts; in fact, the eye becomes progressively more sensitive to short wave (blue)

[8] J. Purkinje, Mag. f.d. gesammte Heilkunde 20, 199 (1826).
[9] P. Reeves, J. Opt. Soc. Am. 1, 148 (1917); P. G. Nutting, Trans. I. E. S. 11, No. 9 939 (1916).

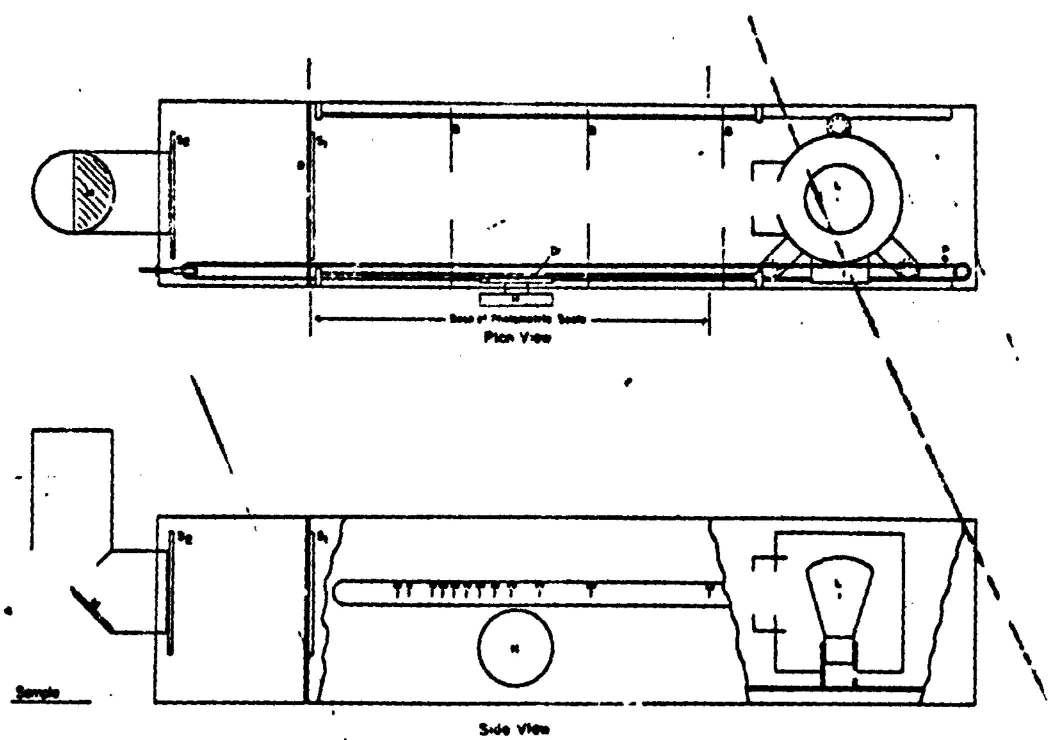


Fig. 2.

and less sensitive to long wave (red) light as the luminance to which it is adapted is reduced from 1000 to about 0.5 microlambert, the rate of change being most pronounced between 200 and 1 microlambert. This shift of the luminosity curve towards shorter wave-lengths explains the Purkinje effect. For adaptation to luminances below 0.5 microlambert, the eye again reaches a steady state, with a constant luminosity curve characteristic of scotopic vision and the eye may be regarded as dark-adapted. We may speak of these luminances as scotopic luminances. In Fig. 1 the adopted luminosity curve of the light-adapted[7] eye and an average curve for the dark-adapted[8] eye are shown. As the luminance of the observed surface is diminished we pass through the region of mesopic vision and the luminosity curve moves progressively from that of the light-adapted eye towards that of the dark-adapted.

[7] A résumé of the data on which the standard ICI luminosity factors are based and of the present status of these factors is given in a paper by Kasson S. Gibson, J. Opt. Soc. Am. 30, 51 (1940).

[8] "Summary of American Opinion on BS/ARP 18. British Standard Specification for Fluorescent and Phosphorescent Paint" prepared for the American Standards Association by L. A. Jones under date of June 15, 1942, gives an average of the luminosity data for low luminances determined by Hecht and Williams and by Weaver.

There are several ways to assign numerical values to a scale of luminance below the photopic region to include mesopic and scotopic luminance.

*A*. Arbitrary application of the photopic luminosity function to all luminances by making visual comparison of fluorescent and phosphorescent materials only with a standard of similar spectral composition, the standard to be evaluated by way of the photopic luminosity function.

*B*. Adoption of an arbitrary photoelectric procedure for evaluating the radiant energy from fluorescent and phosphorescent materials.

*C*. Definition of the unit and scale of mesopic and scotopic luminance in terms of the hypothetical equal-energy source.

*D*. Definition of the unit and scale of mesopic and scotopic luminance in terms of an incandescent lamp operating at 2360°K color temperature.

There have been attempts to make the standard photopic luminosity function (*A*) do. Each material characterized by radiant energy of a new spectral composition, *x*, requires the setting up and evaluation of a new standard. For various samples of the same material, the method gives useful comparisons; but because of the Purkinje effect, photopic luminance of one material compared to another does not necessarily correlate with the observable brightness in the mesopic and scotopic regions of the materials.

A better correlation is obtainable by arbitrary adoption of a photoelectric procedure in which the source-filter photo-cell combination is approximately equivalent to the luminosity function somewhere within the mesopic range (*B*). It is obvious, however, that no one luminosity function can be generally valid.

Procedures *C* and *D* are similar and give values of mesopic and scotopic luminance that correlate well with brightness. The use (*C*) of the hypothetical equal-energy source would bring a unique logical simplicity to the general concept of luminance, but the predominant opinion[8] is that an incandescent-lamp source at 2360°K is more convenient in practice and, on that account, preferable for tentative standardization and use at the present time. By choice the comparison source is assigned mesopic and scotopic luminances by the same methods (inverse-square law, sector-disk relation, aperture relations) used for photopic luminance.

## III. DESCRIPTION OF THE PHOTOMETER

### (a) Photometric Details

The photometer is shown in Fig. 2. The views depart from conventional drawing practice by showing the openings in the baffles and diaphragms as in a section taken through the axis of the beam. Otherwise Fig. 2 shows conventional plan and elevation views.

The lamp illuminates the flashed opal diffusing glass screen, $S_1$, which in turn illuminates a similar screen, $S_2$, the outer surface of which viewed in the front-surface mirror, *M*, forms the comparison field of the photometer. The other or test field is the surface of the sample viewed directly. The luminance of the surface of $S_2$ facing the mirror depends upon the light reaching the surface toward $S_1$, which is closely proportional to the product of the area of the opening in the diaphragm *D* and the luminance of the surface of $S_1$ exposed to $S_2$, since the distance between $S_1$ and $S_2$ is constant. The luminance of the surface of $S_1$ away from the lamp is proportional to the illuminance of the surface toward the lamp which, of course, depends upon the distance between $S_1$ and the lamp.

Let the luminance of *M* as viewed from the sight tube be $B_M$, the reflectance of the mirror be *r* and the luminance of the surface of $S_2$ facing the mirror be $B_2$. We then have that

$$B_M = rB_2.$$

The luminance $B_2$ is proportional to the illuminance $E_2$, on the surface of $S_2$ away from the mirror, or

$$B_2 = k_2E_2,$$

the constant of proportionality, $k_2$, being characteristic of the transmissive properties of $S_2$. Hence

$$B_M = rk_2E_2.$$

$E_2$, however, is equal to the product of the area, *A*, of the opening in the diaphragm *D* and the luminance, $B_1$, of the surface of $S_1$ facing $S_2$, divided by the square of the distance $d_1$, between $S_1$ and $S_2$,

$$E_2 = AB_1/d_1^2,$$

and we accordingly have the expression

$$B_M = rk_2AB_1/d_1^2,$$

for the luminance of the mirror. The luminance $B_1$ is proportional to the illuminance $E_1$ on the surface of $S_1$ toward the lamp, or

$$B_1 = k_1E_1,$$

where $k_1$ is characteristic of the transmissive properties of $S_1$, which gives us

$$B_M = rk_2Ak_1E_1/d_1^2.$$

Finally $E_1 = I/d^2$ where *I* is the luminous intensity of the lamp, *L*, in the direction of $S_1$ and *d* is the distance of *L* from $S_1$, so that

$$B_M = rk_2Ak_1I/d_1^2d^2.$$

In this we see that *r*, $k_2$, $k_1$, *I*, and $d_1$ are fixed characteristics of any particular photometer and can be grouped under a single characteristic, *P*, and hence we can write that

$$B_M = PA/d^2.$$

The photometric scale is graduated from 20 to 0.5, and a set of diaphragms having holes with areas differing by factors of about 10 has been made.[8] These were made with ordinary drills and then calibrated. The photometric scale and the ratios of the areas of the openings in the diaphragms overlap, and values near the ends of the scale can be measured by means of either of two diaphragms. The use of these mechanical means to control the luimnance of the comparison field gives a long range with no change in spectral composition. When the photometer is used without any of the removable diaphragms the maximum reading is more than 150,000 times the minimum reading with the smallest-aperture diaphragm. The opening in the metal plate holding $S_1$ is a limiting diaphragm when none of the removable diaphragms are used.

The field is a plain elliptical field (diametrically divided circle viewed at 45°), the major axis being 1½ inches long. The major axis coincides with the dividing line of the field, which usually is viewed so that the two halves are seen side by side. Since the end of the sight tube is 4 inches from the mirror, the angles subtended at the eye by the field are 15 and 20 degrees for the minor and major axes respectively. The mirror when placed as shown in Fig. 2 serves as a baffle to prevent any light from the screen $S_2$ falling on the test surface.

### (b) Mechanical Details

The mechanical details may, of course, be varied to suit the maker's materials and choice. The photometer used at the Bureau employs the box, track, lamp housing, and scale of a Sharp-Millar[9] photometer. The photometric cube and eyepiece were removed, and the diaphragm-diffusing-screen arrangement described in the previous section installed. Since this type of photometer is no longer commercially available a description of the mechanical details will serve as a guide for anyone wishing to construct one.

The box (Fig. 2) is about 4×4×22 inches. The lamp, $L$, is moved by means of an endless cord which passes over a drum, $Dr$, which is turned by the handwheel, $H$. The sight tube may be turned in its collar to view the test surface at various angles. In order to avoid errors caused by light reflected from the interior of the box a series of baffles, $B$, made of fiber is placed between the lamp and the screen $S_1$, and the interior is painted with a flat (mat) black paint. These baffles are carried on two light rods and are attached to each other and the lamp housing by cords. When the lamp moves toward $S_1$, the housing pushes the baffles successively in front of it; and when it moves away from $S_1$, the cords pull the baffles one after another into their original position. The lamp housing carries an index, $I$, the shadow of which falls upon the translucent scale in the side of the box, and thus there is no parallax. The scale is covered by red plastic to preserve the dark adaptation of the photometric observer. This arrangement also makes it unnecessary to provide a light for reading the scale, which is a great convenience, and avoids the usual scale marked on a space-wasting rod protruding from the box. Since readings are taken in a dark room, no stray light will enter the photometer through the translucent scale. It would be necessary to provide a shutter to cover the translucent scale if readings were taken in a lighted room, the shutter being opened only to read the scale after a setting had been taken.

The lamp housing has a second index on the side opposite the scale index to facilitate the accurate positioning of the filament of the lamp at the unity mark of the photometric scale. The housing runs on a track made of angle brass fastened to the sides of the box. The single wheel has a spring forcing it against the track so that sidewise motion is prevented.

### IV. SUMMARY

A photometer, such as described in this paper makes possible the determination of scotopic and mesopic luminance such as that of fluorescent and phosphorescent materials. The use of a comparison field of color temperature 2360°K and the mechanical control of the luminance of the comparison field is in accord with current American opinion on the datum and method of evaluating luminances in the mesopic and scotopic

[8] Metric drills of 0.5, 1.6, 5, and 16 mm will give areas proportional to 0.25, 2.56, 25, and 256 provided new accurately ground drills are available. For some purposes these area ratios may be sufficiently close to the desired factors of 10.

[9] C. H. Sharp and P. S. Millar, Elec. World 51, 181 (1908); Elec. Rev. 52, 141 (1908); and Electrician 60, 562 (1907–08).

regions. Luminance so evaluated takes the Purkinje effect into account and correlates perfectly with brightness, subjectively evaluated.

The photometer has been used for nearly 3 years in routine measurements of the luminance-time (brightness decay) curve of phosphorescent materials as well as to determine both the luminance and chromaticity of the fluorescent light from papers impregnated or coated with fluorescent chemicals. In measuring the luminance of phosphorescent materials, different observers agree within about 5 percent in the region near 10 effective microlamberts, while the spread between observers increases to about 25 percent when the luminance is in the region of 0.005 effective microlambert. The measurement of fluorescent materials has not been extensive. However, some measure of the effect of color is given by tests of blue fluorescence at about 100 effective microlamberts where four observers made observations within a little less than 25 percent and tests of yellow fluorescence where (with the same observers) the results did not spread by as much as 10 percent at about 200 effective microlamberts. The use for determining chromaticity (where the spectral composition of the comparison source must remain constant while the luminance is varied) has been so satisfactory at low luminances that a photometer has been designed with a much wider range of luminance than the present photometer possesses. This increased range will be adequate for measuring the chromaticity of non-luminescent materials in the photopic range of luminance.

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12.9.

## On the Standard Source for Low Level Photometry

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*Bureau International des Poids et Mesures, Sèvres, Seine-et-Oise, France*
July 25, 1949

THE definitions and agreements of photometry are now well-settled and they received the sanction of the Comité International and the Conférence Générale des Poids et Mesures.[1] Although these definitions, the single official ones today, have been adopted without any limitation of the scope of their validity, they are likely to be modified in the field of low luminance levels, in order to take into account the Purkinje effect, and new international agreements, less simple that the present ones, are likely to be adopted in the future. An almost unanimous agreement has already been achieved at the eleventh session of the C.I.E. (Paris 1948); it settles the choice of the primary standard source for all future photometric system and the magnitude of the unit of what will be, in these systems, the analogy of luminance (or photometric brightness). The text of the resolution adopted unanimously by all the nations (except Australia) which attended the C.I.E., and in particular by the U. S. A., reads as follows:

"Light and Vision. Resolution 2. It is recommended that, when for special purposes the luminous effects of radiant energies of various spectral compositions are evaluated by methods that do not rest on the standard luminosity function adopted by the C.I.E. in 1924, the unit of the quantity corresponding to luminance (photometric brightness) should always be chosen so that this quantity has the numerial value 60 (c.g.s. system) for a black body at the temperature of freezing platinum."

# The Photometry of Colored Light

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Vol. LXII, No. 4, April 1967

A. C. Wall

PRESENT day photometers usually incorporate a photosensitor for evaluating the illumination from an unknown source by comparing it with the illumination from a standard source. The photosensitor must be spectrally corrected to provide a sensitivity curve as close as possible to that of the photopic luminous efficiency function of the CIE standard observer.* This function is the design goal of a photosensitor-filter combination. An example of the goal and the realized spectral response of a particular phototube-filter combination is shown in Fig. 1. (More details will be given below.) The overall match is good and the photosensitor gives excellent results in the photometric measurements of light with a spectral distribution close to that of the light with which the photosensitor is calibrated.

Consider, however, red light transmitted by a filter with a shortwave cutoff at about 630 nanometers and with a transmittance curve as shown in Fig. 2. The output $R$ of the photosensitor is given approximately by

$$R = K \Sigma_\lambda E_\lambda \tau(\lambda) S(\lambda) \Delta\lambda \qquad (1)$$

where $K$ is the proportionality factor, $E_\lambda$ is the spectral irradiance, $S(\lambda)$ is the relative spectral response of the photosensitor, and $\tau(\lambda)$ is the spectral transmittance of the correction filter.

It can be seen from Fig. 2 that the $S(\lambda)\tau(\lambda)$-realized curve is much different from the $S(\lambda)\tau(\lambda)$-goal curve at wavelengths above 630 nanometers. Hence the outputs of the phototube will not have the same constant of proportionality for the illuminations from the red light and the "white" standardizing light.

The investigation reported herein is an evaluation of the adequacy of several photosensitor-filter combinations when used in the measurement of "colored light," which in this paper designates light that has a spectral distribution different from that of the light with which the photometer was calibrated.

## Photosensitors and Correction Filters

The type PJ-14B vacuum phototube, with an S-7 cathode surface, though not available today, is ideally suited to general low-illumination photometry as well as high-illumination flash photometry.[1] It has low

A paper presented at the National Technical Conference of the Illuminating Engineering Society, August 21 to 26, 1966, Minneapolis, Minn. AUTHOR: National Bureau of Standards, Washington, D. C.
*For a two-degree field of view.

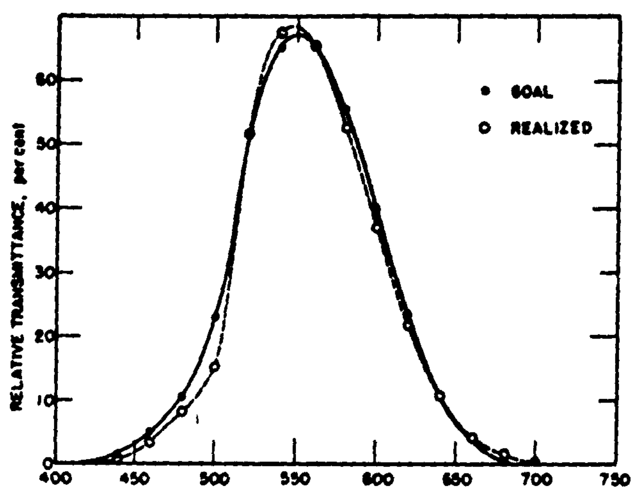


*Figure 1. Spectral transmittance goal of a filter and the transmittance as realized.*

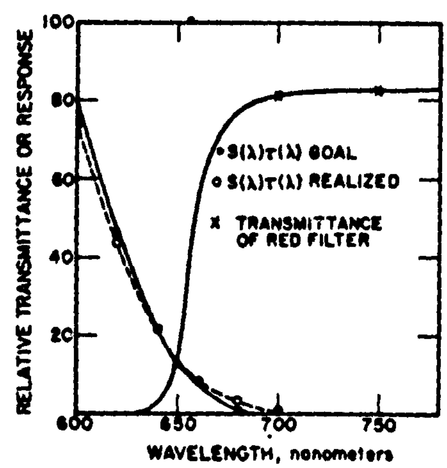


*Figure 2. Relative spectral response goal of a phototube-filter combination, the response as realized, and the spectral transmittance of a filter transmitting red light.*

Table I—Correction Factors for Four Color-Corrected Photosensitors with S-4 Surfaces

| TEST FILTER NUMBER | COLOR DESIGNATION* | LUMINOUS TRANSMITTANCE | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| 1 | Purplish blue | .020 | 1.08 | .97 | .91 | 1.03 |
| 2 | Blue | .039 | 1.10 | 1.02 | .97 | 1.07 |
| 3 | Blue | .102 | .86 | .93 | .88 | .90 |
| 4 | Blue (lunar white) | .166 | .91 | .95 | .91 | .94 |
| 5 | Blue (lunar white) | .246 | .94 | .96 | .95 | .97 |
| 6 | Bluish green | .130 | 1.15 | 1.14 | 1.05 | 1.17 |
| 7 | Green | .198 | 1.14 | 1.10 | 1.03 | 1.14 |
| 8 | Green | .249 | 1.13 | 1.10 | 1.05 | 1.12 |
| 9 | Green | .110 | 1.16 | 1.12 | 1.05 | 1.16 |
| 10 | Green | .047 | 1.17 | 1.15 | 1.07 | 1.18 |
| 11 | Green | .049 | 1.20 | 1.17 | 1.07 | 1.16 |
| 12 | Yellow-green | .405 | 1.16 | 1.14 | 1.05 | 1.15 |
| 13 | Yellowish green | .138 | 1.10 | 1.09 | 1.04 | 1.09 |
| 14 | Yellowish orange | .740 | 1.01 | 1.00 | 1.00 | 1.00 |
| 15 | Yellowish orange | .760 | .98 | .90 | .99 | .95 |
| 16 | Orange | .452 | .94 | .94 | .96 | .92 |
| 17 | Reddish orange | .519 | .90 | .92 | .94 | .90 |
| 18 | Reddish orange | .304 | .81 | .87 | .91 | .84 |
| 19 | Red | .154 | .72 | .80 | .86 | .77 |
| 20 | Red | .106 | .65 | .74 | .80 | .70 |
| 21 | Red | .050 | .46 | .55 | .63 | .52 |
| 22 | Red | .0254 | .33 | .40 | .50 | .42 |

* The chromaticity coordinates are shown in Fig. 3.

dark current and an essentially flat spectral response in the visible region, which makes the design of a correction filter relatively easy.

Optical filters were designed at the National Bureau of Standards for selected tubes of this type to provide phototube-filter combinations with spectral responses which essentially match the CIE standard observer spectral luminous efficiency function $V(\lambda)$. Thus the goal is to design a filter so that

$$K_n \, \tau(\lambda) S(\lambda) = V(\lambda) \qquad (2)$$

where $\tau(\lambda)$ is the spectral transmittance of the filter, $S(\lambda)$ is the relative spectral response of the phototube at wavelength $\lambda$, and $K_n$ is the normalizing factor.

The spectral transmittance of a desired filter was determined by solving Equation (2) for $K_n\tau(\lambda)$ in terms of $V(\lambda)$ and $S(\lambda)$. Having determined the desired function $K_n\tau(\lambda)$, a filter consisting of a suitable combination of four glasses in series was chosen and the spectral transmittance of samples of each glass was determined. From these data the desired thickness of each component was computed. The components were then ground and polished to the desired thicknesses and the spectral transmittances were measured. In those instances in which the measured spectral transmittance did not agree sufficiently well with the desired spectral transmittance, the components were reground to a thickness which gave the desired spectral response.

Fig. 1 shows the spectral response of the desired and the designed phototube-filter, indicating a reasonably good approximation.

The Gillod-Boutry-type phototube is an end-on vacuum phototube with a CsBi cathode surface. The phototube saturates at less than two volts, has a spectral sensitivity that peaks near 500 nm and extends beyond 700 nm, and has low dark current, about $10^{-12}$ ampere. The phototubes were offered to members of the International Bureau of Weights and Measures in 1957 and were subsequently made by a French manufacturer on the basis of orders received. Fifty of the phototubes were purchased by the National Bureau of Standards for precision photometric measurements. J. S. Laufer of NBS designed a filter to correct a spectrally matched group of the tubes so as to have a spectral response which is essentially the CIE spectral luminous efficiency function. The filter design is similar to that described by Nimeroff and Wilson[2] for a sector division filter: the filter is composed of six sectors of varying angular extent, each segment being made of several layers of glass.

The type 929 and the type A29 are vacuum phototubes having an S-4 cathode surface that peaks in response at about 400 nm. The Wratten No. 106 filter is designed to correct a typical phototube with an S-4 surface spectrally to match the CIE luminous efficiency function. The S-4 cathode is found in many of the commercially available photometers, usually with the Wratten No. 106 filter.

The 856 barrier-layer photocell has a selenium photosensitive surface. The cells used incorporate a two-layer spectral correcting filter. Although the manufacturer indicates more than a ten per cent deviation from linearity between 20 to 200 footcandles with a 200-ohm external resistance, Barbrow[3] shows a maximum of 1.4 per cent error between 0.6 and 180 footcandles relative to the reading at 0.6 footcandle with a "zero resistance" circuit for cells of this type.

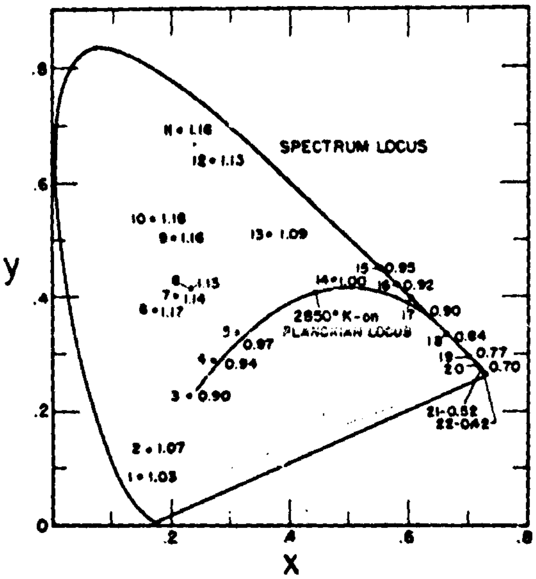


*Figure 3. The 22 filters used plotted on the CIE chromaticity diagram. Shown is the set of correction factors k for one color-corrected S-4 phototube.*

To show that uncorrected photosensitors are useful only for measuring illumination having exactly the same spectral distribution as that of the light with which they are calibrated, measurements were made with the following uncorrected photosensitors: four phototubes with S-4 sensitive surfaces; three phototubes with S-1 sensitive surfaces (peak response at 800 nm); two type PJ-14B tubes; a Gillod-Boutry phototube; and a selenium barrier-layer cell.

## Procedure

The photosensitors were calibrated by exposing them to illumination from a lamp operating at 2854°K; 22 two- by two-inch squares of colored glass, whose spectral transmittance was known from several sets of independent spectrophotometric measurements, were placed in turn in the light beam and readings of the photosensitors were taken. The color designations (see Table I) are those suggested by Kelly.[4]

For each of the 22 colors a correction factor $k$ was calculated, such that

$$k = \tau_s/\tau_m \tag{3}$$

where $\tau_s$ is the transmittance of the filter as determined by a set of spectrophotometric measurements and $\tau_m$ is the transmittance as indicated by two photosensitor readings.

Since
$$E_s = \tau_s E_o, \text{ and} \tag{4a}$$
$$\tau_m = E_m/E_o \tag{4b}$$

where $E_o$ is the measured illuminance on the phototube without the filter, $E_s$ is the "correct" illuminance with the filter, and $E_m$ is the measured illuminance with the filter,

$$E_s = kE_m \tag{5}$$

or, $k$ times the measured illuminance gives the "correct" illuminance which is incident on the phototube.

**Table II—Correction Factors for Color-Corrected Photosensitors**

| TEST FILTER NUMBER | COLOR DESIGNATION* | LUMINOUS TRANSMITTANCE | PHOTOSENSITOR TYPE: PJ-14B 1 | PJ-14B 2 | GILLOD-BOUTRY | BARRIER LAYER 2 | BARRIER LAYER 3 |
|---|---|---|---|---|---|---|---|
| 1 | Purplish blue | .020 | 1.08 | 1.11 | .85 | .93 | .93 |
| 2 | Blue | .039 | 1.05 | 1.05 | .87 | 1.00 | 1.00 |
| 3 | Blue | .102 | .96 | .95 | | 1.01 | 1.02 |
| 4 | Blue (lunar white) | .166 | .97 | .94 | 1.01 | .99 | .99 |
| 5 | Blue (lunar white) | .246 | .97 | .95 | 1.01 | .99 | .99 |
| 6 | Bluish green | .130 | .99 | .92 | .96 | .97 | .99 |
| 7 | Green | .198 | .99 | .92 | .99 | .97 | 1.00 |
| 8 | Green | .249 | .99 | .93 | .99 | .98 | 1.00 |
| 9 | Green | .110 | .98 | .89 | .99 | .97 | .98 |
| 10 | Green | .047 | .99 | .87 | .98 | .94 | .98 |
| 11 | Green | .049 | .98 | .86 | .98 | | .98 |
| 12 | Yellow-green | .405 | 1.00 | .95 | | 1.03 | 1.06 |
| 13 | Yellowish green | .138 | .99 | .90 | 1.00 | .98 | 1.00 |
| 14 | Yellowish orange | .740 | .99 | .99 | | 1.02 | 1.04 |
| 15 | Yellowish orange | .760 | .96 | .98 | 1.02 | .98 | .99 |
| 16 | Orange | .652 | .98 | 1.01 | 1.04 | .99 | .99 |
| 17 | Reddish orange | .519 | 1.00 | 1.06 | 1.05 | 1.00 | 1.00 |
| 18 | Reddish orange | .304 | 1.00 | 1.10 | 1.02 | .99 | .96 |
| 19 | Red | .134 | .97 | 1.12 | 1.01 | .99 | .93 |
| 20 | Red | .106 | .94 | 1.12 | .98 | .99 | .92 |
| 21 | Red | .050 | .79 | .98 | | .89 | .83 |
| 22 | Red | .0254 | .67 | .88 | | .88 | .79 |

* The chromaticity coordinates are shown in Fig. 3.

**Table III—Correction Factors for Uncorrected Photosensitors**

| | COLOR DESIGNATION* | LUMINOUS TRANSMITTANCE | S-4 SURFACES | | | | BARRIER LAYER | S-1 SURFACES | | | PJ-14B | | GILLOD-BOUTRY |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | | 1 | 2 | 3 | 1 | 2 | |
| 1 | Purplish blue | .020 | .08 | .10 | .08 | .09 | .22 | .59 | .51 | .53 | .32 | .42 | .11 |
| 2 | Blue | .039 | .14 | .16 | .14 | .15 | .34 | .58 | .52 | .55 | .44 | .52 | .18 |
| 3 | Blue | .102 | .26 | .28 | .25 | .27 | .35 | .13 | .14 | .13 | .27 | .18 | .31 |
| 4 | Blue (lunar white) | .166 | .37 | .41 | .37 | .39 | .66 | .22 | .22 | .22 | .40 | .28 | .45 |
| 5 | Blue (lunar white) | .246 | .48 | .52 | .48 | .50 | .76 | .31 | .32 | .31 | .52 | .40 | .55 |
| 6 | Bluish green | .130 | .45 | .46 | .43 | .45 | .81 | 6.67 | 7.43 | 8.13 | 1.34 | 2.31 | .32 |
| 7 | Green | .198 | .52 | .53 | .30 | .52 | [illegible] | 8.61 | 6.39 | 7.62 | 1.46 | 2.53 | .59 |
| 8 | Green | .249 | .36 | .38 | .34 | .36 | .92 | 7.66 | 5.47 | 6.92 | 1.53 | 2.57 | .65 |
| 9 | Green | .110 | .53 | .55 | .52 | .54 | .96 | 8.80 | 8.80 | 8.46 | 1.57 | 2.62 | .63 |
| 10 | Green | .047 | .46 | .49 | .46 | .47 | .90 | 7.84 | 7.84 | 7.52 | 1.47 | 2.40 | .55 |
| 11 | Green | .049 | .96 | .93 | .96 | .93 | 1.29 | 19.60 | 12.30 | 16.33 | 2.23 | 3.83 | .98 |
| 12 | Yellow-green | .405 | 1.08 | 1.02 | 1.06 | 1.04 | 1.25 | 4.74 | 4.09 | 4.62 | 1.81 | 2.65 | 1.08 |
| 13 | Yellowish green | .138 | .99 | .98 | 1.01 | .98 | 1.30 | 15.33 | 10.33 | 13.33 | 2.19 | 3.73 | .06 |
| 14 | Yellowish orange | .740 | 1.23 | 1.21 | 1.24 | 1.22 | 1.08 | 1.44 | 1.47 | 1.45 | 1.11 | 1.22 | 1.14 |
| 15 | Yellowish orange | .760 | 2.08 | 1.89 | 2.18 | 2.03 | 1.13 | .93 | .94 | .94 | 1.04 | .99 | 1.37 |
| 16 | Orange | .652 | 2.43 | 2.20 | 2.66 | 2.35 | 1.07 | .77 | .77 | .77 | .92 | .84 | 1.64 |
| 17 | Reddish orange | .519 | 2.75 | 2.48 | 3.09 | 2.71 | 1.00 | .62 | .63 | .63 | .79 | .70 | 1.65 |
| 18 | Reddish orange | .304 | 3.21 | 3.02 | 4.16 | 3.52 | .81 | .37 | .37 | .37 | .53 | .44 | 1.52 |
| 19 | Red | .154 | 3.28 | 3.21 | 4.91 | 2.98 | .64 | .19 | .20 | .20 | .31 | .24 | 1.22 |
| 20 | Red | .106 | 2.94 | 3.03 | 4.49 | 3.41 | .56 | .13 | .14 | .13 | .23 | .16 | 1.04 |
| 21 | Red | .050 | 2.17 | 2.27 | 3.60 | 2.60 | .43 | .07 | .07 | .07 | .12 | .09 | .71 |
| 22 | Red | .0234 | 1.69 | 2.03 | 3.30 | 2.44 | .36 | .04 | .04 | .04 | .07 | .05 | .53 |

* The chromaticity coordinates are shown in Fig. 3.

## Results

The $k$ values for one photosensitor with a color-corrected S-4 surface are given in Fig. 3; $k$ values for all four color-corrected S-4 surfaces are given in Table I. Correction factors between 0.81 and 1.20 are applicable to all colors except the reds. Factors for phototube No. 3 are within ±0.05 of 1.00 for 11 colors, while the other three phototubes are within ±0.05 for only three or four colors.

The PJ-14B phototube No. 1 is within ±0.08 of 1.00 for all colors except the two deepest reds and within ±0.05 for all but the two reds and the purplish blue, as shown in Table II. PJ-14B phototube No. 2 shows better correction for the reds than No. 1; correction in the blues and greens is less adequate, however. (The average correction factor for both phototubes is 0.97). The Gillod-Boutry is within ±0.05 for all but two of the 17 filters for which measurements were made.

The $k$ values for the noncolor-corrected photosensitors are given in Table III.

## Discussion

The data for the barrier-layer type of photocells show that, when they are calibrated with light at a color temperature of 2854°K, many of the colors shown in the tables can be measured with small errors. Also, more tellingly in the reds, if the photocell is calibrated with a filter close on the chromaticity diagram to the color being measured, the error is reduced. For example, a photocell calibrated with light passing through filter No. 19 could be used for measuring light through filter No. 20 within one per cent (on the basis of the data of barrier-layer cell No. 3).

Since many of the correction factors of Table I for adjacent filters are within a few per cent of each other, photosensitors with S-4 surfaces and Wratten No. 106 filters can also be calibrated with light spectrally close to the light being measured for increasing accuracy.

Each set of correction factors of Tables I through III applies only to a specific photosensitor and not to the type in general. The factors do, however, suggest the magnitude of the errors that might be expected when measuring the intensity of light of the color indicated.

Although the PJ-14B phototube is no longer available commercially, there is a comparable replacement, the type Z-1454, which is, however, too expensive for general photometric use. The Gillod-Boutry phototube is included as an example of a phototube for precision measurements, which has been made, and could, in the future, be made available to a group of interested purchasers at a reasonable price.

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Journal of Research of the National Bureau of Standards Vol. 62, No. 4, April 1959 Research Paper 2946

# Absolute Light-Scattering Photometer: I. Design and Operation

12.11.

Donald McIntyre and G. C. Doderer [1]

A new light-scattering photometer has been designed and built for determining the absolute scattering from polymer solutions. The instrument is also capable of performing as a research instrument for making measurements at very low and very high angles, and at very low and very high intensities of scattered light. The instrument scans the angular scattering either manually or automatically while measuring continuously the ratio of the scattered light to the incident light.

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Vol. 68A, No. 1, January February 1964

# An Absolute Light Scattering Photometer: II. Direct Determination of Scattered Light From Solutions

Donald McIntyre

12.12.

(August 9, 1963)

The light scattering photometer recently described in this journal by McIntyre and Doderer has been examined to determine its ability to measure the absolute scattering of liquids. The absolute scattering of polymer solutions was determined from transmission measurements and from two different transverse measurements. The experimental results are in good agreement. The variables of the photometric system were also analyzed and experimentally studied to determine its ability to measure absolute scattering of liquids under different geometrical arrangements.

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